13th Algebraic Hyperstructures and its Applications Conference
July 24-27, 2017 Istanbul/TURKEY
ABSTRACT BOOK

Editors
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http://www.aha2017.yildiz.edu.tr
13th Algebraic Hyperstructures and its Applications Conference (AHA2017), organized by the International Algebraic Hyperstructures Association will take place from 24th July to the 27th July 2017 in Istanbul, a fascinating city built on two Continents, divided by the Bosphorus Strait and this is one of the greatest cities in the world where you can see a modern western city combined with a traditional eastern city, it’s a melting pot of many civilizations and different people.

The series of International Conferences on Algebraic Hyperstructures and Applications (AHA) aims at bringing together researchers and academics for the presentation and discussion of novel theories and applications of Algebraic. The conference covers a broad spectrum of topics related to Algebraic Hyperstructures.

AHA2017 provides an ideal academic platform for researchers and scientists to present the latest research findings in mathematics. The conference aims to bring together leading academic scientists, researchers and research scholars to exchange and share their experiences and research results about mathematics and engineering studies.

We would like to thank to Yildiz Technical University for their invaluable supports. We would also like to thank to all contributors to conference, especially to keynote speakers who share their significant scientific knowledge with us, to organizing and scientific committee for their great effort on evaluating the manuscripts. We do believe and hope that each contributor will get benefit from the conference.

We hope to see you in 14th Algebraic Hyperstructures and its Applications Conference (AHA2020) at Romania.

Yours Sincerely,

Prof. Dr. Bayram Ali ERSOY

Chair of AHA2017
Previous AHA Events

- 1978, held in Taormina, Italy (November 25-28) Organized by P.Corsini
- 1983, held in Taormina, Italy (October 21-24) Organized by P.Corsini
- 1985, held in Udine, Italy (October, 15-18) Organized by P.Corsini
- 1990, held in Xanthi, Greece (June 27-30) Organized by T.Vougiouklis
- 1993, held in Iasi, Romania (July 4-10) Organized by M.Stefanescu
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- 2002, held in Samothrace, Greece (September 1-9) Organized by T.Vougiouklis
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- 2008, held in Brno, South Moravia, Czech Republic (September 3-9) Organized by S.Hoskova
- 2011, held in Pescara, Italy (October, 16-21) Organized by A. Maturo
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Some Applications of Algebraic Hyperstructures

Bijan Davvaz

In this study, we describe the applications of algebraic hyperstructures and survey related works. Hyperstructures represent a natural extension of algebraic structures and they were introduced in 1934 by F. Marty. He generalized the notion of groups by defining hypergroups. Algebraic hyperstructures have many applications in various sciences. In [1], Corsini and Leoreanu presented some of the numerous applications of algebraic hyperstructures, especially those from the last fifteen years, to the following subjects: geometry, hypergraphs, binary relations, lattices, fuzzy sets and rough sets, automata, cryptography, codes, median algebras, relation algebras, artificial intelligence and probabilities. The largest class of hyperstructures is the one that satisfies weak axioms, i.e., the non-empty intersection replaces the equality. These are called Hv-structures and they were introduced in 1990 by Vougiouklis [2]. The latter hyperstructures have many applications to different disciplines like Biology, Chemistry, Physics, and so on. In several papers, Davvaz et al. [3-9] introduced some chemical examples of hyperstructures. For instance, algebraic hyperstructures associated to chain reactions; algebraic hyperstructures associated to dismutation reactions; algebraic hyperstructures associated to redox reactions; hyperstructures associated to electrochemical cells. Another motivation for the study of hyperstructures comes from biology. In [10], the main objective of authors is to provide examples of hyperstructures associated to inheritance. They explored the algebraic hyperstructure that naturally occurs as genetic information gets passed down through generations. Mathematically, the algebraic hyperstructures that arise in genetics are very interesting ones. They are generally commutative and weakly associative. Moreover, many of the algebraic properties of these hyperstructures have genetic significance. Indeed, there is an interplay between the purely algebraic hyperstructures and the corresponding genetic properties, that makes the subject so fascinating. The examples given in [10] indicated that the theory of genetic hyperstructure algebras is generally worth practicing. Mendel, the father of genetics took the first steps in defining “contrasting characters, genotypes in \( F_1 \) and \( F_2 \ldots \) and setting different laws”. The

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genotypes of $F_2$ is dependent on the type of its parents genotype and it follows certain roles. In [11], the authors analyzed the second generation genotypes of monohybrid and a dihybrid with a mathematical structure. They used the concept of Hv-semigroup structure in the $F_2$-genotypes with cross operation and proved that this is an Hv-semigroup. They also determined the kinds of number of the Hv-subsemigroups of $F_2$-genotypes. In [12], the authors provided examples about different types of inheritance (Mendelian and Non-Mendelian inheritance) and relate them to hyperstructures and generalize the work done in [10]. The feature of hyperstructures allows us to extend this theory into the elementary particle physics. In [13], the authors have considered one important group of the elementary particles, Leptons. They have shown this set that along with the interactions between its members can be described by the algebraic hyperstructure. In [14], Asghari-Larimia and Davvaz presented a connection between algebraic hyperstructures and number theory. They introduced a hyperoperation associated to the set of all arithmetic functions and analyzed the properties of this new hyperoperation. Several characterization theorems are obtained, especially in connection with multiplicative functions. Then, Al Tahan and Davvaz [15] constructed a hyperring structure on the set of arithmetic functions.

**Keywords:** hyperstructure, chemistry, biology, physics, number theory.

**2010 AMS Classification:** 20N20

**References:**
5. Davvaz, B. and Dehghan Nezhad, A., Dismutation reactions as experimental verifications of ternary algebraic hyperstructures, MATCH Communications in  

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The Hv-matrix Representations

Thomas Vougiouklis
Emeritus Professor

The Theory of Representations of Hyperstructures was started in mid 80’s but that time there was not any general definition of hyperfield. The Hv-structures, were introduced in 4th AHA Congress 1990, and at the same time, the general definition of the hyperfield, was given. Since then the Theory of Representations is refereed mainly on Hv-groups by Hv-matrices, that is that, the matrices have entries elements of an Hv-field or from an Hv-ring. In Hv-structures the weak axioms replace the classical axioms of structures by replacing the ‘equality’ by the ‘non empty intersection’. The characteristic property of Hv-structures, is that a partial order on Hv-structures on the same underline set, is defined. The weak properties increase extremely the number of hyperstructures defined in the same set, therefore it is reasonable to find applications in mathematics and in other applied sciences, as well. On the other side, in order to obtain strict results, we ask from applied sciences to give more axioms and more restrictions. This is the case, for example, in nuclear physics with Santilli’s iso-theory. In representation theory the researchers have to treat well almost all the classical algebraic structures from semigroups to Lie-algebras. We present the problems, some new results and we give to researchers open problems in mathematics from hyperstructures.

Keywords: Hyperstructures, Hv-structures, Hv-matrix.


References:


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Some Categorical Aspects of Algebraic Hyperstructures

Reza Ameri

In this study we briefly discuss some of algebraic hyperstructures theory in view point of category theory and present some features of the various hyperstructures such as hypergroups, hyperrings, hypermodules and etc. In this regards we investigate various categories of hyperstructures based on various kinds of morphisms, especially on multivalued homomorphisms. We will proceed by introducing some categorical objects such as, zero object, product, coproduct and free objects. Finally, we constructs some functors from categories of hyperstructures to the correspondence classical algebraic category.

Today hyperstructures rapidly developed in view point of theory and application and many concept of classical algebra are appear in this theory. In parallel to this progress main questions will rise about and some terminology has been used improper. On the other hands, some of terminology may be bad used. Also, for study the relationships between hyperstructures and classical algebra we need to use the exact language to correct mathematically descriptions of these notions. For example in hyperstructures theory we use the phrases such as: a hypergroup is a generalization of a group, the class of polygroup is a generalization of ordinary group. The fundamental relation on a hypergroup is a function which assign to each hypergroup a group or in general to every hyperalgebra one can assign an algebra via the fundamental relation. Here naturally give rise some main questions:

What is different between the class of hypergroups and groups?
Are they really mathematically different?

And many other questions which appears to study of algebraic hyperstructures and its relationship to the related algebraic structures. In this paper we briefly to mention the role of category theory as a useful tools to answer to these questions as well as we introduce some categorical objects such as product, kernel, free and etc. in category of Krasner hypermodules.

Keywords: category, hypergroups, hypermodules, fundamental functor, hyperadditive category.

2010 AMS Classification: 03G99, 06B99, 06F05.
Acknowledgements:
The author partially has been supported by "Algebraic Hyperstructure Excellence (AHETM), Tarbiat Modares University, Tehran, Iran" and "Research Center in Algebraic hyperstructures and Fuzzy Mathematics, University of Mazandaran, Babolsar, Iran".

References:
4. SM. Anvariye, S. Mirvakili, B. Davvaz, Fundamental relation on (m; n)- hypermodules over (m; n)-hyperrings, Ars combin. 94(2010)273-288.
17. S. Mirvakili, B. Davvaz, Constructions of (m; n)-hyperrings. MATEMAT. 67,1(2015)1-16.

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Hyperstructures and some of the most recent applications

Piergiulio Corsini

After a brief history of Hypergroups, since the beginning around the 40s till today, one gives an excursus of the most recent applications of this topic to Fuzzy Sets and Chinese groups as HX-hypergroups.

Keywords: Hyperstructures

2010 AMS Classification: 20N20

References:

Weak Closure operations on ideals of a BCK-algebra

Hashem Bordbar\textsuperscript{1} and Mohammad Mehdi Zahedi\textsuperscript{2}

Weak closure operation, which is more general form than closure operation, on ideals of BCK-algebras is introduced, and related properties are investigated. Regarding weak closure operation, finite type, (strong) quasi-primeness, tender and naive are considered. Using a weak closure operation \textquoteleft cl\textprime \textquoteright and an ideal A of a lower BCK-semilattice X with the greatest element 1, a new ideal K of X containing the ideal A\textsuperscript{cl} of X is established. Using this ideal K, a new function

\[ \text{cl}_t : I(X) \to I(X); A \to K \]

is given, and related properties are considered. We show that if \textquoteleft cl\textprime \textquoteright is a tender (resp., naive) weak closure operation on I(X), then so are \textquoteleft cl\textprime t\textquoteright and \textquoteleft cl\textprime f\textquoteright.

**Keywords:** closure operation, (finite type, tender, naive) weak closure operation, zeromeet element, meet ideal.

**2010 AMS Classification:** 06F35, 03G25.

**References:**

EXTENSION OF POLYGROUPS BY POLYGROUPS VIA FACTOR POLYGROUPS

Lumnije Shehu\textsuperscript{1}, Hani Khashan\textsuperscript{2}

The idea of constructing extensions of polygroups via factor polygroups comes from an extension that De Salvo introduced in [13] which is called \((H,G)\)-hypergroups. Basically, given a hypergroup \((H,+)\) and mutually disjoint sets \(\{A_i\}_{i \in G}\) where \(G\) is a given group such that \(A_0 = H\). Set \(K = \bigcup_{i \in G} A_i\) and define a hyper operation \(\oplus\) on \(K\) as follows: For all \(x, y \in H\), \(x \oplus y = x + y\). For all \(x \in A_i\) and \(y \in A_j\) such that \(A_i \times A_j \neq H \times H\), \(x \oplus y = A_k\) where \(i + j = k\). This extension of \(G\) by \(H\) represents a hypergroup. The wreath product \(H[G]\) introduced in [2] can be obtained by De Salvo’s construction when \(H\) and \(G\) are polygroups, \(A_i = H\) and \(A_i = \{i\}\) for \(i \neq 0\). In our construction, we consider two polygroups \(H\) and \(L\). We restrict the cardinalities of sets \(A_i\), \(i \neq 0\) to be equal to the cardinality of some factor polygroup \(H/I\) and the cardinality of \(A_0\) equals to that of \(H\). The hyper operation on \(K = \bigcup_{i \in L} A_i\) is based on the hyper operations on the factor polygroup \(H/I\) and the polygroup \(L\). In principle, the element zero of \(L\) is enlarged by the polygroup \(H\) and the rest of the elements of \(L\) are enlarged by isomorphic copies of the factor polygroup \(H/I\). This construction yields a polygroup in the case when the subpolygroup \(I\) is normal. However, the kernel of a strong homomorphism is not necessarily normal, [10]. Therefore, by weakening the condition of normality, we obtain the utmost possible extensions. Indeed, we define and study regularly normal subpolygroups. After introducing the isomorphism theorems subject to these subpolygroups, we are able to present our new extension via factor polygroups.

Keywords: hypergroups, polygroups, polygroups extensions, regularly normal subpolygroups.

2010 AMS Classification: 20N20

References:


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On crossed polysquares and fundamental relations

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In this paper, we introduce the notion of crossed polysquare of polygroups and we give some of its properties. Our results extend the classical results of crossed squares to crossed polysquares. One of the main tools in the study to polygroups is the fundamental relations. These relations connect polygroups to groups, and on the other hand, introduce new important classes. So, we consider a crossed polysquare and by using the concept of fundamental relation, we obtain a crossed square.

Keywords: Crossed module, crossed square, polygroup, fundamental relation.

2010 AMS Classification: 13D99, 20N20, 18D35

References:
12- R. Brown and N. D. Gilbert, Algebraic models of 3-types and automorphism structures

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Recent Advances in EL-hyperstructures

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\textit{EL-hyperstructures} are semihypergroups or ring-like hyperstructures $S$ constructed from partially (or, in many cases, quasi-) ordered semigroups, where the hyperoperation on $S$ is defined by $a \ast b = [a \cdot b]_{\leq} = \{x \in S | a \cdot b \leq x\}$ for all $a, b \in S$. When looking for examples, one can construct numerous EL-semihypergroups, hypergroups, join spaces, lattice-like or ring-like hyperstructures in a number of natural contexts as the set $S$ can be a number domain, set of words of a given alphabet, set of objects, properties of which can be described by numbers, sets of vectors or matrices, etc. The relations can be numerous as well: ordering numbers (or numerically described properties) by size, divisibility relation, or relations motivated by some special contexts. EL-hyperstructures were introduced by Chvalina in [1] and named so and studied by Novák in e.g. [2,3].

In our paper we focus on some recent advances in the area of EL-hyperstructures. We clarify the issue of antisymmetry of the relation “\leq” and include examples when it is a quasi-ordering which is moreover symmetric, i.e. an equivalence. We show the use of the construction in the area of lattice-like hyperstructures, i.e. for $H_v$-semilattices, hypersemilattices or hyperlattices (which were studied in [4]). We discuss implications of extensivity of the hyperoperation, i.e. contexts when $\{a, b\}$ is included in $a \ast b$ for all $a, b \in S$. We also mention the way EL-hyperstructures can be used to construct Cartesian composition of multiautomata. Finally, we briefly mention the relation of EL-hyperstructures to some other concepts of hyperstructure theory, where the idea of ordering is used, such as ordered hyperstructures, quasi-order hypergroups or some special cases of BCI-algebras.

\textbf{Keyword(s):} EL-hyperstructures, hyperstructure theory, ordered semigroups, quasi-ordered semigroup

\textbf{2010 AMS Classification:} 20N20, 06F05

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HH*-Intuitionistic Heyting Valued $\Omega$-Algebra

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Intuitionistic Logic was introduced by L. E. J. Brouwer and Heyting algebra was defined by A. Heyting in 1930, to formalize the Brouwer’s intuitionistic logic. The concept of Heyting algebra has been accepted as the basis for intuitionistic propositional logic. Heyting algebras have had applications in different areas. The co-Heyting algebra is the same lattice with dual operation of Heyting algebra. Also, co-Heyting algebras have several applications in different areas.

In this paper, we introduced the new concept HH*- Intuitionistic Heyting Valued $\Omega$-Algebra. The purpose of introducing this new concept is to expand the field of researchers’ area using both membership degree and non-membership degree. This allows us to get more sensitive results. The concepts of HH*- Intuitionistic Heyting valued set, HH*- Intuitionistic Heyting valued relation, HH*- Intuitionistic Heyting valued $\Omega$-algebra and the homomorphism over HH*- Intuitionistic Heyting valued $\Omega$-algebra were defined.

Keyword(s): Heyting Valued Algebra, co-Heyting Valued Algebra, Omega Algebra, Intuitionistic Logic.

2010 AMS Classification: 03C05

Reference(s):

This study was supported by the Research Fund of Mersin University in Turkey with Project Number: 2015-TP3-1249.
Algebraic Approach to Multiplicative Set

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Rough set theory was introduced by Pawlak in 1982. The theory of rough sets is an extension of set theory as a subset of a universe is defined by a pair of ordinary sets called the lower and upper approximations. The algebraic structures of rough sets were studied by several authors. Davvaz introduced the notion of rough subring in 2004. Some properties of the lower and the upper approximations in a ring were examined by Davvaz.

In this study, we examined some relations between rough sets and multiplicative subsets of a commutative ring $R$. The lower and upper approximations of the set $X$ with respect to "~" were defined and algebraic properties were examined.

**Keyword(s):** Rough set, Rough subring, Multiplicative set, Fuzzy ideal.

**2010 AMS Classification:** Primary 05C38, 15A15; Secondary 05A15, 15A18

**References:**


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19. Zadeh L.A., The concept of linguistic variable and its applications to approximate reasoning,
   Part I, Inform. Sci. 8 (1975) 199-249;
   Part II, Inform. Sci. 8 (1975) 301-357;
   Part II, Inform. Sci. 9 (1976) 43-80;

This study was supported by the Research Fund of Mersin University in Turkey with Project Number: 2015-TP3-1249.

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On the relation between categories of \((m, n)\)-ary hypermodules and \((m, n)\)-ary modules

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We introduce the category of \(R_{(m,n)}\)-hypermodules over a Krasner \((m, n)\)-hyperring \(R\) and obtain some categorical objects in this category such as product and coproduct. We apply the fundamental relations \(\varepsilon^*\) and \(\Gamma^*\) on, \(M\) and, \(R\) respectively to construct fundamental functor from the category of \(R_{(m,n)}\)-hypermodules into category of \(R/\Gamma^*\)-modules. In particular we consider the fundamental relation on \((m, n)\)-hypermodules, and construct functor from the category of \((m, n)\)-hypermodules to the category of \((m, n)\)-modules. Then, we find the relations between hom, product, coproduct and fundamental functor.

In this section we give some definitions and results of \(n\)-ary hyperstructures which we need in what follows.

A mapping \(f : H \times H \times \ldots \times H \to P^*(H)\) is called an \(n\)-ary hyperoperation, where \(P^*(H)\) is the set of all non-empty subsets of \(H\).

**Definition 2.3** [11] A Krasner \((m, n)\)-hyperring is algebraic hyperstructure \((R, h, k)\) which satisfies the following axioms:

- \((R, h)\) is a canonical \(m\)-ary hypergroup;
- \((R, k)\) is an \(n\)-ary semigroup;
- the \(n\)-ary operation \(k\) is distributive to the \(m\)-ary hyperoperation \(h\), i.e., for all \(a_i^{m-1}, a_i^n, x_i^m \in R\), and \(1 \leq i \leq n,

\[
k(a_i^{m-1}, h(x_i^n), a_i^n) = h(k(a_i^{m-1}, x_i^n), a_i^n,..., k(a_i^{m-1}, x_m^n), a_i^n));
\]

- \(0\) is a zero element (absorbing element), of the \(n\)-ary operation \(k\), i.e., for \(x_2^n \in R\) we have \(k(0, x_2^n) = k(x_2^n, 0, x_3^n) = \ldots = k(x_2^n, 0)\).

**Definition 2.5** [3] A Krasner \((m, n)\)-hypermodule \((M, f, g)\) is an \((m, n)\)-hypermodule with a canonical \(m\)-ary hypergroup \((M, f)\) over a Krasner \((m, n)\)-hyperring \((R, h, k)\).

**Definition 2.8** [11] Let \((R, h, k)\) be \((m, n)\)-hyperring. The relation \(\Gamma^*\) is the smallest equivalence relation such that the quotient \((R/\Gamma^*, h/\Gamma^*, k/\Gamma^*)\) be \((m, n)\)-ring, where \(R/\Gamma^*\) is the set of equivalence classes. The \(\Gamma^*\) is called fundamental equivalence relation.

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Various categories of \((m, n)\)-ary hypermodules

**Definition 3.2** The category \(R_{(m,n)} - Hmod\) of \((m, n)\)-ary hypermodules defined as follows:

- the objects of \(R_{(m,n)} - Hmod\) are \((m, n)\)-hypermodules,
- for the objects \(M\) and \(K\), the set of all morphisms from \(M\) to \(K\) is defined as follows:
  \[\text{Hom}_R(M, K) = \{f \mid f: M \to P^s(K)\text{ is an }m\text{-homomorphism}\}\],
- the composition \(gf\) of morphisms \(f: M \to P^s(K)\) and \(g: K \to P^s(L)\) defined as follows:
  \[gf: H \to P^s(K),\quad gf(x) = \bigcup_{t \in f(x)} g(t),\]
- for any object \(H\), the morphism \(1_H: H \to P^s(H)\), defined by \(1_H(x) = \{x\}\), is the identity morphism.

**Theorem 3.5** \(F: R_{(m,n)} - hmod \to R_{(m,n)}/\Gamma^* - mod\) defined by \(F(M) = M/\epsilon^*\) and \(F(\phi) = \phi^*\), is a functor \(\phi: M_1 \to M_2\) and \(\phi^*: M_1/\epsilon^* \to M_2/\epsilon^*\), where \(R_{(m,n)}/\Gamma^* - mod\) is the category of all \((m, n)\)-modules over \(R/\Gamma^*\).

**Remark 3.9** In the following of this paper we consider the category of all \((m, n)\)-hypermodules over a \((m, n)\)-hyperring \(R\), in the sense of Krasner \((m, n)\)-hypermodules over commutative Krasner \((m, n)\)-hyperring \(R\) with identity. We denote this category by \(R_{(m,n)} - KHmod\). Hence, the objects of \(R_{(m,n)} - KHmod\) are Krasner \((m, n)\)-hypermodules over commutative Krasner \((m, n)\)-hyperring with identity and all morphisms are multivalued homomorphisms.

In this section, concepts of direct hyper product and direct hyper coproduct of a Krasner \((m, n)\)-hypermodule are defined. Also we give some properties of the category \(R_{(m,n)} - KHmod\).

**Definition 4.3** Let \(\{M_i \mid i \in I\}\) be a family of \((m, n)\)-hypermodules. We define a hyperoperation on \(\prod_{i \in I} M_i\) as follows:

\[F(a_{i_1}^{im}) = \{t_i \mid t_i \in f_i(a_{i_1}^{im}) \{a_{i_1}^{im}\} \in \prod_{i \in I} M_i\}\].

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For \( r \in R \) and \( a_i \in \prod_{i \in I} M_i \), define
\[
G(i^{(n-1)}\{a_i\}_{i \in I}) = \left\{ g_i(i^{(n-1)}(a_i)) \right\}_{i \in I}.
\]
then \( \prod_{i \in I} M_i \), together with \( m \)-ary hyperoperation \( F \) and \( n \)-ary operation \( G \) is called direct hyper product \( \{M_i | i \in I\} \).

**Definition 4.6** The direct hyper sum of the family \( \{M_i | i \in I\} \) of \((m, n)\)-hypermodules, denoted by \( \bigoplus_{i \in I} M_i \), is the set of all \( \{a_i\}_{i \in I} \), where \( a_i \) can be non-zero only for a finite number of indices.

**Theorem 4.11** Let \( \{M_i | i \in I\} \) be a family of \((m, n)\)-hypermodules over an \((m, n)\)-ary hyperring \( R \) and let \( \varepsilon^*_{M_i}, i \in I \) and \( \varepsilon^*_{\bigoplus_{i \in I} M_i} \) be fundamental equivalence relation on \( M_i \) and \( \bigoplus_{i \in I} M_i \) respectively. Then

\[
\phi_1 : (\prod_{i \in I} M_i) / \varepsilon^*_{\bigoplus_{i \in I} M_i} \cong \prod_{i \in I} M_i / \varepsilon^*_{M_i},
\]
and
\[
\phi_2 : (\prod_{i \in I} M_i) / \varepsilon^*_{\bigoplus_{i \in I} M_i} \cong \prod_{i \in I} M_i / \varepsilon^*_{M_i}.
\]

**Theorem 4.12** Fundamental functor \( F \) preserves zero object, product and coproduct.

**Proposition 5.3** Let \( \{M_i | i \in I\} \) be a family of \((m, n)\)-hypermodules over an \((m, n)\)-ary hyperring \( R \) and \( N \) also is an \((m, n)\)-hypermodule and \( F \) be fundamental functor. Then
\[
F(\text{hom}_R(\bigoplus_{i \in I} M_i, N)) \cong \prod_{i \in I} (F(\text{hom}_R(M_i, N))).
\]

**Corollary 5.5** Let \( A, B, C \) be \((m, n)\)-hypermodules over an \((m, n)\)-ary hyperring \( R \) and \( F \) be fundamental functor. Then isomorphism
\[
F(\text{hom}_R(A \bigoplus B, C)) \cong F(\text{hom}_R(A, C)) \bigoplus F(\text{hom}_R(B, C))
\]
is natural.
Keywords: category, $(m,n)$-hypermodules, product, coproduct, additive category.

References


Soft Bi-Ideals of Soft LA-Semigroups

Yıldıray Çelik

In this paper, we present notion of soft bi-ideal of a soft ring and give some results on it. Also, we introduce concept of soft bi-ideal of a soft LA-semigroup and investigate some properties of it.

Keywords: soft ring, soft bi-ideal, soft LA-semigroup

2010 AMS Classification: 06F05, 16D25

Reference(s):
Sequences of Groups and Hypergroups of Linear Ordinary Differential Operators

Jan Chvalina¹, Michal Novákov², David Staněk³

Linear ODE’s are a classical tool for constructing many useful models for description of numerous processes. Given their standard forms, left-hand sides of such equations (both of homogeneous and non-homogeneous) are called linear differential operators. Groups of such operators of different orders can be constructed and some of their properties studied. This includes solvability, relation to quasi-automata or actions on specific spaces or structures.

Using a suitable ordering or quasi-ordering of groups of linear differential operators we construct hyperstructures of linear differential operators. Then, with the help of these hyperstructures, we construct multiautomata which are cardinal sums of perfect semisimple submultiautomata.

The main objective of our paper is to focus on the study of sequences (finite or countable) of groups and hypergroups of linear differential operators of decreasing orders. For this we use inclusion embeddings and group homomorphisms. In particular, we obtain and study what we call “coupled sequences”. We also include a construction of sequences of second-order linear differential operators in the Jacobi form, i.e. such operators that the coefficient at the first-order derivative is zero. Our results can be generalized to operators of an arbitrary order.

We also apply our considerations to the theory of (multi-)automata. In particular, we obtain actions of abelian groups and hypergroups constructed from linear spaces of polynomials of various dimensions over additive abelian groups of differential operators of the corresponding order with constant coefficients at the highest-order derivatives.

Keyword(s): hyperstructure theory, linear differential operators, ODE, theory of automata

2010 AMS Classification: 20N20, 68Q70, 47D03

Reference(s):
On Different Approach of Fuzzy Ring Homomorphims

Umit DENİZ

In this study we approach the definition of $TL$–ring homomorphism. In literature the definition of fuzzy ring homomorphism is given by using the classic functions. In this study we give the definition of fuzzy ring homomorphism by using the definition of Mustafa Demirci’s fuzzy function. Some definition and theorems of ring homomorphism in classic algebra is adapted to fuzzy algebra and proved.

Keywords: Fuzzy Functions, Fuzzy Equivalence Relations, Triangular Norms, Fuzzy Subrings, Fuzzy Ideals

2010 AMS Classification: Mathematics

References:

\(\mathcal{L}\)-hyperstructures

Mahmood Bakhshi

In this paper, as a generalization of familiar classical ordered algebraic structures such as ordered semigroups and ordered groups the notion of \(\mathcal{L}\)-hyperstructure is introduced. Giving some examples it is shown that the familiar ordered hyperstructures and also those hyper algebraic structures arose from logic can be viewed as a special types of \(\mathcal{L}\)-hyperstructres. After that investigating basic properties, some types of hyperideals are introduced, thier properties are investigated and some characterizations and the connections among them are obtained.

**Keyword(s):** hyperstructure, ordered sets, algebras of logics

**2010 AMS Classification:** 20N20, 06F15, 06F35, 06D35

1. Main results

**Definition 2.1.** By a *language of hyperstructures* we mean a set \(\mathcal{L}\) consists of a set \(\mathcal{R}\) of relation symbols and a set \(\mathcal{F}\) of set-valued function symbols such that to each member of \(\mathcal{R}\) (of \(\mathcal{F}\)) is associated a natural number (a non-negative integer) called the *arity* of the symbol. \(\mathcal{F}_n\) denotes the set of set-valued function symbols in \(\mathcal{F}\) of arity \(n\), and \(\mathcal{R}_n\) denotes the set of relation symbols in \(\mathcal{R}\) of arity \(n\).

**Definition 2.2.** Let \(\mathcal{L}\) be a language. An ordered pair \(\mathcal{A} = \langle A; \mathcal{L} \rangle\) in which \(A\) is a non-empty set and \(\mathcal{L}\) consists of a family \(\mathcal{R}\) of fundamental relations on \(A\) indexed by \(\mathcal{R}\) and a family \(\mathcal{F}\) of fundamental hyper operations on \(A\) indexed by \(\mathcal{F}\) is called a hyperstructure of type \(\mathcal{L}\) (or \(\mathcal{L}\)-hyperstructure). \(A\) is called the universe of \(\mathcal{A}\). When \(\mathcal{R} = \emptyset\), \(A\) is a hyper algebra and if \(\mathcal{F} = \emptyset\), \(A\) is a relational structure. If \(\mathcal{L}\) is finite, say \(\mathcal{F} = \{f_1, \ldots, f_m\}\) and \(\mathcal{R} = \{r_1, \ldots, r_n\}\), we often write \(\langle A; f_1, \ldots, f_m; r_1, \ldots, r_n \rangle\) instead of \(\langle A; \mathcal{L} \rangle\). If \(f_i \in \mathcal{F}_n\) is an \(n_i\)-ary function symbol and \(r_j \in \mathcal{R}_l\) is an \(l_j\)-ary relation we write, for brevity, \(\mathcal{L}_{<n_1, \ldots, n_m;l_1, \ldots, l_k>}-\)hyperstructure, where \(n_1 \geq n_2 \geq \cdots \geq n_m\) and \(l_1 \geq l_2 \geq \cdots \geq l_k\), instead of \(\mathcal{L}\)-structure. In this paper, we focus on \(\mathcal{L}_{<2;2>\text{-hyperstructures}}\); for brevity, whenever it is clear from the context, we write \(\mathcal{L}\)-hyperstructure instead of \(\mathcal{L}_{<2;2>\text{-hyperstructure}}\).

**Definition 2.3.** Let \(\langle H;\mathcal{V};\leq \rangle\) be an \(\mathcal{L}\)-hyperstructure. The relation \(\leq\) can be eneralized to nonempty subsets of \(H\) as follows: for \(A, B \in P^*(H)\),

(i) \(A \leq_w B\), if there exist \(a \in A\) and \(b \in B\) such that \(a \leq b\),

(ii) \(A \leq_{rw} B\), if for each \(b \in B\) there exists \(a \in A\) such that \(a \leq b\),

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(iii) \( A \leq_{tw} B \), if for each \( a \in A \) there exists \( b \in B \) such that \( a \leq b \).
(iv) \( A \leq_{tw} B \), if \( A \leq_{lw} B \) and \( A \leq_{rw} B \).
(v) \( A \leq_{s} B \) if for each \( a \in A \) and \( b \in B \) we have \( a \leq b \).

**Definition 2.4.** Let \( < H; \circ, \leq > \) be an \( \mathcal{L} \)-hyperstructure. We say that \( \leq \) is

(i) **weak left (right) compatible** if
\[
x \leq y \Rightarrow a \circ x \leq_{w} a \circ y \quad (x \circ a \leq_{w} y \circ a) \quad \forall a \in H
\]
If \( \leq \) is weak left and weak right compatible it is said to be **weak compatible**.

(ii) **r-left (r-right) compatible** if
\[
x \leq y \Rightarrow a \circ x \leq_{rw} a \circ y \quad (x \circ a \leq_{rw} y \circ a) \quad \forall a \in H
\]
If \( \leq \) is r-left and r-right compatible it is said to be **r-compatible**.

(iii) **l-left (l-right) compatible** if
\[
x \leq y \Rightarrow a \circ x \leq_{lw} a \circ y \quad (x \circ a \leq_{lw} y \circ a) \quad \forall a \in H
\]
If \( \leq \) is l-left and l-right compatible it is said to be **l-compatible**.

(iv) **t-left (t-right) compatible** if \( \leq \) is l-left and r-left compatible (l-right and r-right compatible).
If \( \leq \) is t-left and t-right compatible it is said to be **t-compatible** or briefly **compatible**.

(v) **strong left (right) compatible** if
\[
x \leq y \Rightarrow a \circ x \leq_{s} a \circ y \quad (x \circ a \leq_{s} y \circ a) \quad \forall a \in H
\]
If \( \leq \) is strong left and strong right compatible it is said to be **strong compatible**.

**Definition 2.5.** Let \( \leq \) be any types of the relations introduced in Definition 2.3. We say that \( \leq \) is **reversed left (right) compatible** if
\[
x \leq y \Rightarrow a \circ y \leq a \circ x \quad (resp. \ y \circ a \leq x \circ a) \quad \forall a \in H
\]

**Definition 2.6.** By a (weak, l-, r-, t-, strong) \( \mathcal{L} \)-hyperstructure we mean an \( \mathcal{L} \)-hyperstructure on which is defined a (weak, l-, r-, t-, strong) compatible binary relation.

**Remark 2.7.** For convenience, we drop the prefix two-sided and so a two-sided \( \mathcal{L} \)-hyperstructure is called an \( \mathcal{L} \)-hyperstructure.

**Definition 2.8.** An \( \mathcal{L} \)-hyperstructure \( < H; \circ, \leq > \) in which \( \circ \) is commutative (associative) is said to be a **commutative \( \mathcal{L} \)-hyperstructure** (resp, **\( \mathcal{L} \)-semihypergroup**).

**Definition 2.9.** An element \( e \) of an \( \mathcal{L} \)-hyperstructure \( < H; \circ, \leq > \) is called an **identity** if
\[
x \in x \circ e \cap e \circ x, \quad \forall x \in H.
\]

**Example 2.10.**
(i) Any hyper \( K \)-algebra [] and any hyper \( MV \)-algebra [] is an \( \mathcal{L}_{<2,2>} \)-hyperstructure in which the binary relation satisfies Definition 2.3(i).

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(ii) Any hyper residuated lattice \([\cdot]\) is an \(L_{<2;2>}\)-hyperstructure in which the binary relation is weak right compatible with respect to the multiplication and weak left compatible with respect to the residuation.

(iii) Any hyper \(BC\)\(K\)-algebra \([\cdot]\) is an \(L_{<2;2>}\)-hyperstructure in which the binary relation is reversed \(l\)-left compatible.

(iv) Any ordered semihypergroup \([\cdot]\) is an \(L_{<2;2>}\)-hyperstructure with an \(l\)-left compatible relation.

(v) Consider \(\mathbb{R}_1 = [1, \infty)\), the set of all real numbers greater than 1, as a poset with the natural ordering, and define \(x \circ y\) to be the set of all upper bounds of \(\{x, y\}\). Thus \(< \mathbb{R}_1; \circ, \leq >\) is a commutative \(r\)-\(L_{<2;2>}\)-semihypergroup with 1 as the unique identity.

(vi) Let \(< G; \ast, e, \leq >\) be an ordered group, and let \(x \circ y = < \{x, y\} >\), the subgroup of \(G\) generated by \(\{x, y\}\). Then \(< G; \circ, \leq >\) is a commutative \(L_{<2;2>}\)-hyperstructure.

(vii) Let \(< L; \lor, \land, 0 >\) be a lattice with the least element 0. For \(a, b \in L\), let \(a \circ b = F(a \land b)\), where \(F(x)\) is the principal filter generated by \(x \in L\). Then, \(< L; \circ >\) is a commutative \(r\)-\(L\)-hyperstructure.

(viii) Let \(H = \{a, b\}\) be a chain with \(a < b\). We define a hyperoperation \(\circ\) on \(H\) as in Table 1. Then, \(< H; \circ, \leq >\) is an \(r\)-\(L\)-hyperstructure, whereas it is not \(1\)-\(L\)-hyperstructure because \(a \leq b\) but \(a \circ a \not\leq \text{tw} a \circ b\). Indeed, \(b \in a \circ a\) but there is not any element \(x \in a \circ b\) such that \(b \leq x\).

<table>
<thead>
<tr>
<th>(\circ)</th>
<th>(a)</th>
<th>(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>{a, b}</td>
<td>{a}</td>
</tr>
<tr>
<td>(b)</td>
<td>{a}</td>
<td>{a, b}</td>
</tr>
</tbody>
</table>

**Table 1: Cayley table of Example 2.10(viii)**

**Definition 2.11.** By an ordered \(L\)-hyperstructure we mean an \(L\)-hyperstructure in which the binary relation is a partial ordering.

**Definition 2.14.** Let \(H\) be a (weak, left, right, strong) ordered \(L\)-hyperstructure. A down set \(I\) of \(H\) is called

(i) left hyperideal if \(HI \subseteq I\),

(ii) right hyperideal if \(IH \subseteq I\).

If \(A\) is a left and a right hyperideal, it is called a hyperideal of \(H\).

**Example 2.15.**

(i) Consider the ordered \(L\)-hyperstructure \(< z, \circ, \leq >\) given in Example 2.10(vi). It is not difficult to check that the only hyper ideal of \(z\) is itself.
(ii) Let $H = \{a, b, c\}$ be a partially ordered set, where $a < b$ and define hyperoperation `$\circ$' on $H$ as shown in Table 2. Then $< H, \circ, \leq >$ is an ordered $\mathcal{L}$-semihypergroup in which $I = \{a, b\}$ is a hyperideal of $H$.

<table>
<thead>
<tr>
<th>$\circ$</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>${a, b}$</td>
<td>${a}$</td>
<td>${a}$</td>
</tr>
<tr>
<td>b</td>
<td>${a}$</td>
<td>${b}$</td>
<td>${b}$</td>
</tr>
<tr>
<td>c</td>
<td>${a}$</td>
<td>${b}$</td>
<td>${c}$</td>
</tr>
</tbody>
</table>

Table 2: Cayley table of Example 2.15(ii)

(iii) Consider the partially ordered set $H$ given in part (ii). We define a hyperoperation on $H$ as in Table 3. Then $< H, \circ, \leq >$ is an ordered $\mathcal{L}$-semihypergroup in which $I = \{a, b\}$ is a left hyperideal of $H$ but it is not a right hyperideal because $a \circ c = \{a, b, c\} \not\subseteq I$. Thus $IH \not\subseteq I$.

<table>
<thead>
<tr>
<th>$\circ$</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>${a, b}$</td>
<td>${a}$</td>
<td>${a, b, c}$</td>
</tr>
<tr>
<td>b</td>
<td>${a}$</td>
<td>${b}$</td>
<td>${c}$</td>
</tr>
<tr>
<td>c</td>
<td>${a, b}$</td>
<td>${a, b}$</td>
<td>${a, b, c}$</td>
</tr>
</tbody>
</table>

Table 3: Cayley table of Example 2.15(iii)

(iv) Consider the partially ordered set $H$ given in part (ii), again. Then, $< H, \circ, \leq >$ is an ordered $\mathcal{L}$-semihypergroup in which the hyperoperation is given as in Table 4. It is easy to check that $I = \{a, b\}$ is a right hyperideal of $H$ which is not a left hyperideal because $c \circ b = \{c\} \not\subseteq I$. Hence, $HI \not\subseteq I$.

<table>
<thead>
<tr>
<th>$\circ$</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>${a, b}$</td>
<td>${a}$</td>
<td>${a, b}$</td>
</tr>
<tr>
<td>b</td>
<td>${a}$</td>
<td>${b}$</td>
<td>${a, b}$</td>
</tr>
<tr>
<td>c</td>
<td>${a, b, c}$</td>
<td>${c}$</td>
<td>${a, b, c}$</td>
</tr>
</tbody>
</table>

Table 4: Cayley table of Example 2.15(iv)

Reference(s):


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New connections between hyperstructures and Graph Theory

Masoumeh Golmohamadian\(^1\), Mohammad Mehdi Zahedi\(^2\)

In present study, we investigate the relation of dominating sets in graphs and hyperstructures. We introduce different hyperoperations and semihypergroups, deriving from dominating sets and minimal dominating sets of a graph and we examine their properties.

A vertex \(v\) in a graph \(G\) is said to dominate itself and each of its neighbors and a set \(D\) of vertices of \(G\) is a dominating set of \(G\) if every vertex of \(G\) is dominated by at least one vertex of \(D\). Let \(G = (V,E)\) be a graph, \(H_i\) be a dominating set and \(H\) be the set of all dominating sets of \(G\). Then we define \(\theta(H_i)\) as the maximum number of vertices of \(H_i\), that we can omit from \(H_i\) to convert it to a minimal dominating set. For every \(H_i, H_j \in H\), we define the commutative semihypergroup \((H,\ast)\) in the following way:

\[
H_i \ast H_j = \begin{cases} 
H_i, & \text{if } \theta(H_i) < \theta(H_j) \\
H_j, & \text{if } \theta(H_j) < \theta(H_i) \\
H_i, & \text{if } \theta(H_i) = \theta(H_j) \text{ and } |H_i| < |H_j| \\
H_j, & \text{if } \theta(H_i) = \theta(H_j) \text{ and } |H_i| < |H_j| \\
\{H_i, H_j\}, & \text{if } \theta(H_i) = \theta(H_j) \text{ and } |H_i| = |H_j|
\end{cases}
\]

In addition, we construct another commutative semihypergroup \((H,o)\) by considering \(\lambda(H_i)\) as the set of all minimal dominating sets which have minimum cardinality among all minimal dominating sets that are obtained from \(H_i\).

We also make a connection between minimal dominating sets and hyperstructures. Let \(S\) be the set of all minimal dominating sets and \(S_i\) be a minimal dominating set. Then we define \(\phi(S_i)\) by

\[
\phi(S_i) = \text{the number of isolated vertices of } G[S_i] / |S_i|
\]

We introduce the commutative semihypergroup \((S,\ast_i)\) as follows:

for every \(S_m, S_n \in S\)

\[
S_m \ast_i S_n = \begin{cases} 
S_n, & \text{if } \phi(S_m) < \phi(S_n) \\
S_m, & \text{if } \phi(S_m) > \phi(S_n) \\
\{S_m, S_n\}, & \text{if } \phi(S_m) = \phi(S_n)
\end{cases}
\]

We investigate some situations in which this semihypergroup is hypergroup and give some examples to clarify them. Finally, we present a new class of graphs in which this semihypergroup will be a hypergroup.

**Keyword(s):** Dominating sets in graphs: Minimal dominating sets in graphs: Semihypergroup: Hypergroup

**2010 AMS Classification:** 05C69, 20N20

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References:

Ideals in HvMV-algebras

Mahmood Bakhshi, Roghayeh Taherpoor and Akbar Paad

In this paper, first some basic definitions are reviewed. Then some types of ideals such as Hv MV-ideals, weak Hv MV-ideals and nodal Hv MV-ideals are introduced and some characterizations and their properties are obtained.

Introduction

In 1958, Chang [3], introduced the concept of an MV-algebra as an algebraic proof of the completeness theorem for \( \aleph_0 \)-valued Łukasiewicz propositional calculus. After that many mathematicians have worked on MV-algebras and obtained significant results. Mundici [7] proved that MV-algebras and Abelian A-groups with strong unit are categorically equivalent. After that Marty [6] introduced the notion of a hypergroup several authors worked on hypergroups, especially in France and in the United States, but also in Italy, Russia and Japan. Bakhshi et al. introduced ordered polygroups [2] which are subclasses of hypergroups on which is defined a partial ordering with special property. Hyperstructures have many applications to several sectors of both pure and applied sciences. A short review of the theory of hyperstructures appear in [4]. Vougiouklis [8] introduced a generalization of the well-known algebraic hyperstructures such as hypergroup so-called Hv-structures. Actually some axioms concerning the above hyperstructures such as the associative law, the distributive law and so on are replaced by their corresponding weak axioms. In order to obtain a suitable generalization of MV-algebras which may be equivalent (categorically) to a certain subclass of the class of Hv -groups, the author introduced the concept of an Hv MV-algebra [1] and obtained some related results.

HvMV-algebras: Basic properties

In this section, the concept of an HvMV-algebra is introduced. For more details we refer to thereferences.

Definition 2.1. An Hv MV-algebra is a nonempty set \( H \) endowed with a binary hyperoperation ‘\( \oplus \)’, a unary operation ‘\( * \)’ and a constant ‘\( 0 \)’ satisfying the following conditions:

- \( (HvMV1) \) \( x \oplus (y \oplus z) \cap (x \oplus y) \oplus z \neq \emptyset \), (weak associativity)
- \( (HvMV2) \) \( x \oplus y \cap y \oplus x \neq \emptyset \), (weak commutativity)
- \( (Hv MV3) \) \( (x*)* = x \),

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(Hv MV4) \((x* \oplus y)* \oplus y \cap (y* \oplus x)* \oplus x \neq \emptyset\).

(HvMV5) \(0* \in x \oplus 0* \cap 0* \oplus x\).

(Hv MV6) \(0* \in x \oplus x* \cap x* \oplus x\), (Hv MV7) \(x \in x \oplus 0 \cap 0 \oplus x\),

(Hv MV8) \(0* \in x \oplus y \cap y \oplus x\) and \(0* \in y* \oplus x \cap x \oplus y*\) imply \(x = y\).

**Remark 2.2.** On any Hv MV-algebra \(H\), a binary relation ‘\(\preceq\)’ by

\[x \preceq y \iff 0* \in x* \oplus y \cap y \oplus x*\]

is introduced which is reflexive and antisymmetric but not necessarily transitive.

**HvMV-ideals**

In this section, the ideal theory of HvMV-algebras is studied. The concepts of weak HvMV-ideal and HvMV-ideal are introduced and some properties and fundamental results are obtained.

**Definition 3.1.** Let \(I\) be a nonempty subset of \(H\) satisfying \((I_0)\) \(x \preceq y\) and \(y \in I\) imply \(x \in I\).

Then, \(I\) is called

1. an HvMV-ideal if \(x \oplus y \subseteq I\), for all \(x, y \in I\),
2. a weak HvMV-ideal if \(x \oplus y \preceq I\), for all \(x, y \in I\).

**Theorem 3.2.** Every Hv MV-ideal is a weak Hv MV-ideal.

**Theorem 3.4.** A non empty subset \(I\) of \(H\) is a weak HvMV-ideal if and only if it satisfies \((I_0)\) and \((x \oplus y) \cap I \neq \emptyset\), for all \(x, y \in I\).

**Theorem 3.7.** If \(\{I_\alpha : \alpha \in \Lambda\}\) is a nonempty family of Hv MV-ideals of \(H\), then \(\bigcap_{\alpha \in \Lambda} I_\alpha\) is a HvMV-ideal.

**Definition 3.8.** Let \(A\) be a nonempty subset of \(H\) and \(\{I_\alpha : \alpha \in \Lambda\}\) be a family of Hv MV-ideals of \(H\) containing \(A\). Then \(\bigcap_{\alpha \in \Lambda} I_\alpha\) is called the Hv MV-ideal generated by \(A\), denoted by \((A)\).

**Remark 3.12.** It seems that Theorem 3.7 does not hold for weak Hv MV-ideals, in general. At this point, we can’t find any example showing this (open problem). But, there is a situation in which we know that the intersection of a family of weak Hv MV-ideals is again a weak Hv MV-ideal (see Theorem 3.17). When the same result holds for weak Hv MV-ideals, the weak Hv MV-ideal generated by a family \(A\) of weak Hv MV-ideals of \(H\) is denoted by \((A)_w\).

Let Hv MVI (WHv MVI) denotes the set of all Hv MV-ideals (weak Hv MV-ideals) of \(H\). Then, HvMVI (WHv MVI) together with the set inclusion, as a partial ordering, is a poset.

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Theorem 3.13. (HvMVI, ⊆) is a complete lattice and if WHvMVI is closed with respect to the intersection, HvMVI is a complete sublattice of the complete lattice (WHvMVI, ⊆).

Definition 3.14. An element a ∈ H is called a right scalar if for all x ∈ H, |x ⊕ a| = 1, i.e., the set x ⊕ a is singleton. We denote the set of all right scalars of H by R(H).

Definition 3.16. A subset S of H is said to be ⊕-closed, if for all x, y ∈ S, x ⊕ y ⊆ S.

Theorem 3.17. Assume that |x ⊕ y| < ∞, for all x, y ∈ H, ≼ is transitive and monotone and R(H) is ⊕-closed. If A is a nonempty subset of H contained in R(H), then

\[(A)_w = \{ x \in H : x \preceq ((a_1 \oplus a_2) \oplus \ldots \oplus a_n), \text{ for some } n \in \mathbb{N}, a_1, \ldots, a_n \in A \} \].

Particularly, if A = \{a\}, \((A)_w = \{ x \in H : x \preceq na, \text{ for some } n \in \mathbb{N} \} \].

Nodal HvMV-ideals

Definition 4.1. Let H be an HvMV-algebra. By a (weak) nodal HvMV-ideal of H we mean a (weak) Hv MV-ideal of H which is comparable with each (weak) HvMV-ideal of H.

Example 4.2. Consider the HvMV-algebra (H; ⊕, *, 0), where ⊕ and * are defined as in Table 4. Routine calculations show that the only HvMV-ideals of H are \{0\}, \{0, a\} and H. So, every Hv MV-ideal of H is a nodal Hv MV-ideal.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>{0}</td>
<td>{0, a}</td>
<td>{0, b}</td>
<td>{0, c}</td>
<td>{0, a, b, c, 1}</td>
</tr>
<tr>
<td>a</td>
<td>{0, a}</td>
<td>{0, a}</td>
<td>{0, a, b, c, 1}</td>
<td>{0, a, b, c, 1}</td>
<td>{0, a, b, c, 1}</td>
</tr>
<tr>
<td>b</td>
<td>{0, b}</td>
<td>{0, a, b, c, 1}</td>
<td>{0, a, b, c, 1}</td>
<td>{0, a, b, c}</td>
<td>{0, a, b, c, 1}</td>
</tr>
<tr>
<td>c</td>
<td>{0, c}</td>
<td>{0, a, b, c, 1}</td>
<td>{0, a, b, c, 1}</td>
<td>{0, a, b, c}</td>
<td>{0, a, b, c, 1}</td>
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<td>1</td>
<td>{0, a, b, c, 1}</td>
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<td>{0, a, b, c, 1}</td>
<td>{0, a, b, c, 1}</td>
<td>{0, a, b, c, 1}</td>
</tr>
</tbody>
</table>

* 1   b   a   c   0

Table 4: Cayley table of H, MV-algebra given in Example 4.2

Proposition 4.5. Any nodal Hv MV-ideal is a nodal weak Hv MV-ideal.

Theorem 4.6. Let I be a (weak) HvMV-ideal of H. If for every x ∈ I and for every y ∈ H \I, x ≼ y, then I is a nodal (weak) HvMV-ideal.
Theorem 4.7. In an HvMV-algebra with a totally ordered, any(weak)HvMV-ideal is a nodal(weak)HvMV-ideal.

Theorem 4.8. Assume that the conditions of Theorem 3.17 holds for HvMV-algebra H. If \( x \in R(H) \) is a node, \( (x)_\omega \) is also a nodal weak Hv MV-ideal of H.

For weak Hv MV-ideal I of H and \( x \in H \), the weak Hv MV-ideal of H Generated by \( I \cup \{x\} \) will denoted by \( I(x) \).

Theorem 4.10. Assume that the conditions of Theorem 3.17 holds for HvMV-algebra H. If I is a nodal weak Hv MV-ideal of H and \( x \in R(H) \) is a node, then \( I(x) \) is a nodal weak Hv MV-ideal of H.

Theorem 4.11. The intersection of any nonempty family of nodal HvMV-ideals is again a nodalHvMV-ideal.

Considering Theorem 3.13 we get

Corollary 4.12. Let \( N(H) \) be the set of all nodal Hv MV-ideals of H. Then \( (N(H), \subseteq) \) is a complete sublattice of \( (HvMVI, \subseteq) \).

Keyword(s): hyperstructures, algebras of logics.

2010 AMS Classification: 06F35, 03G25.

Reference(s):
Overview on the Height of a Hyperideal in Krasner Hyperrings

Hashem Bordbar\textsuperscript{1}, Irina Cristea\textsuperscript{2}, and Michal Novak\textsuperscript{3}

Similarly as in ring theory, the notion of height of a prime hyperideal of a hyperring has recently been defined and studied [1], extending the concept of dimension of a hyperring, in the context of commutative Krasner hyperrings. These are hyperstructures endowed with an additive hyperoperation and a multiplicative operation, satisfying certain properties, introduced by Krasner [7] as a tool in the approximation of valued fields. The height of a proper prime hyperideal of a Krasner hyperring is defined as the maximum of the lengths of the chains of distinct prime hyperideals contained in it, or it is $\infty$ if such a number does not exist.

One of the most significant theorems in commutative algebra is called Krull’s height theorem or Krull’s principal ideal theorem. Using the properties of prime hyperideals in Noetherian Krasner hyperrings, we present an extension of this theorem [1]: If $R$ is a commutative Krasner hyperring and $I$ is a proper principal hyperideal of $R$, then the height of a minimal prime hyperideal of $R$ over $I$ is at most 1. Later on in [2], we extended this result, proving that in a commutative Krasner hyperring $R$, the height of a minimal prime hyperideal over a proper hyperideal of $R$ generated by $n$ elements is at most $n$. The converse of this theorem is also true.

Our future goal is to extend these results to other classes of hyperrings, highlighting their differences/similarities with the classical results for commutative rings.

Keywords: Krasner hyperring, prime/maximal hyperideal, Noetherian hyperring, height of a prime hyperideal, dimension of a hyperring

2010 AMS Classification: 06F35, 03G25.

Reference(s):
1. H. Bordbar, I. Cristea, Height of prime hyperideals in Krasner hyperrings, Filomat (accepted for publication in 2017).

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E-mails: bordbar.amirh@gmail.com
7. M. Krasner, Approximation des corps values complets de caracteristique p; p > 0, par ceux de caracteristique zero, Colloque d’Algebre Superieure (Bruxelles, Decembre 1956), CBRM, Bruxelles, 1957.
Theory of Double-framed soft set theory on Hyper BCK-algebra

Hashem Bordbar$^1$ and Young Bae Jun$^2$

The notion of double-framed soft (strong) hyper BCK-ideal of hyper BCK-algebra is introduced, and related properties are investigated. Characterization of double-framed soft (strong) hyper BCK-ideal is considered, and relation between double-framed soft hyper BCK-ideal and double-framed soft strong hyper BCK-ideal is discussed.

**Keywords:** Cubic intuitionistic set, cubic intuitionistic ideal, positive implicative cubic intuitionistic ideal.

**2010 AMS Classification:** 06F35, 03G25, 06D72.

**References:**

On P-hopes and P-$H_v$-structures on the plane

Achilles Dramalidis$^1$, Ioanna Iliou$^2$

In this paper we deal with P-hyperstructures which are defined in $H_v$-groups. Using a weak commutative hyperoperation on the plane and a specific subset of this plane we construct various P-$H_v$-structures. In addition, we study the existence of units and inverses of these constructions, connecting them with Join Spaces, as well.

**Keyword(s):** Hyperstructures, $H_v$-structures, hopes, P-hyperstructures.

**2010 AMS Classification:** 20N20

**References:**


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On Neutrosophic Linear Spaces

Banu Pazar Varol 1, Vildan Çetkin 2, Halis Aygün 3

Smarandache introduced the neutrosophy which is a branch of philosophy. Then Wang et.al. defined single valued neutrosophic sets. Neutrosophic set is a part of neutrosophy which studies the origin, nature and scope of neutralities. In neutrosophic set, truth-membership, indeterminacy membership and false-membership functional values are independent. Single valued neutrosophic set is applied to algebraic and topological structures. In this paper, we introduce neutrosophic linear space over the neutrosophic field and consider its main properties.

Keywords: Neutrosophic set, single valued neutrosophic set, linear space

2010 AMS Classification: 08A72, 06D72

References:

The Relationships between the Orders Induced by Implications and Uninorms

M. Nesibe Kesicioğlu

In this paper, an order by means of implications on a bounded lattice possessing some special properties is defined and some of its properties are discussed. By giving an order based on uninorms on a bounded lattice, the relationships between such generated orders are investigated.

Keywords: Implications, partial order, bounded lattice, law of importation

2010 AMS Classification: 03E72, 03B52

References:


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A Survey on Order-equivalent Uninorms

M. Nesibe Kesicioğlu¹, Ümit Ertugrul², Funda Karaçal³

In this paper, an equivalence on the class of uninorms on a bounded lattice \( L \) based on the equality of the orders is discussed. Some relationships between the orders induced by t-norms and their N-dual t-conorms are determined. Also, defining the set of all incomparable elements w.r.t. the order induced by uninorms, some relationships with the sets of all incomparable elements w.r.t. the orders induced by the corresponding underlying t-norm and t-conorm are presented.

**Keywords:** Uninorm, bounded lattice, partial order, equivalence of uninorms

**2010 AMS Classification:** 03E72, 03B52

**References:**


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THE REFLECTOR Functor AND THE LATTICE $\mathbb{L}(\mathcal{R})$

Cerbu Olga$^1$ and Dumitru Botnaru$^1$

In the category $\mathcal{C}_2\mathcal{V}$ of locally convex topological vector spaces [RR] we examine a class of factorization structures for which the reflector functor transforms the class of projections or the class of injections, or both classes into themselves. Such functors are usually studied (see [K], [G], [B], [BC], [C]) and appear when studying the semireflexive subcategories [CB].

Let $\Pi$ be the subcategory of the complete spaces with the weak topology and $\pi: \mathcal{C}_2\mathcal{V} \to \Pi$ - the reflector functor. The subcategory $\Pi$ is the minimal element in the lattice $\mathbb{R}$.

Let $\mathcal{R} \in \mathbb{R}$. For every object $X$ of the category $\mathcal{C}_2\mathcal{V}$ let be $r^X: X \to rX$ and $\pi^X: X \to \pi X$ the $\mathcal{R}$ and $\Pi$-repliques. Since $\Pi \subset \mathcal{R}$, we have $\pi^X = v^X r^X$ for a morphism $v^X$. We denote $\mathcal{U} = \mathcal{U}(\mathcal{R}) = \{ v^X \mid X \in \mathcal{C}_2\mathcal{V} \}$, $\mathcal{V} = \mathcal{V}(\mathcal{R}) = \{ v^X \mid X \in \mathcal{C}_2\mathcal{V} \}$. We have the following factorization structures:

$$(\mathcal{P}', \mathcal{I}') = (\mathcal{P}'(\mathcal{R}), \mathcal{I}'(\mathcal{R})) = (\mathcal{V}^-, \mathcal{V}^-), (\mathcal{P}', \mathcal{I}) = (\mathcal{P}'(\mathcal{R}), \mathcal{I}'(\mathcal{R})) = (\mathcal{U}^-, \mathcal{U}^-).$$

For $\mathcal{R} \in \mathbb{R}$ we denote by $\mathbb{L}(\mathcal{R})$ the class of factorization structures $(\mathcal{E}, \mathcal{M})$, for which $\mathcal{P}'(\mathcal{R}) \subset \mathcal{E} \subset \mathcal{P}\,'(\mathcal{R})$ and $\mathcal{L}_u(\mathcal{R}) = \{ (\mathcal{E}, \mathcal{M}) \in \mathbb{L}(\mathcal{R}) \mid \mathcal{M} \subset \mathcal{M}_u \}$ (see [B]), where $\mathcal{M}_u$ is a class of the universal monomorphisms (see [B]).

$$\mathcal{I}'_u = \mathcal{I}'_u(\mathcal{R}) = (\mathcal{E}_p \cup \mathcal{U}(\mathcal{R})) \downarrow, \mathcal{P}'_u = \mathcal{P}'(\mathcal{R}) = (\mathcal{I}'_u)\downarrow.$$

**Definition 1.** Let $r: \mathcal{C} \to \mathcal{R}$ be a covariant functor, and $(\mathcal{P}, \mathcal{I})$ - a factorization structure (the left or right factorization structure). We say that this functor $r$ is:

1. P-functor, if $r(\mathcal{P}) \subset \mathcal{P}$.
2. I-functor, if $r(\mathcal{I}) \subset \mathcal{I}$.
3. $(\mathcal{P}; \mathcal{I})$-functor, if $r(\mathcal{P}) \subset \mathcal{P}$ and $r(\mathcal{I}) \subset \mathcal{I}$.

**Proposition 2.** 1. $\mathbb{L}_u(\mathcal{R})$ is a complete lattice with the minimal element $(\mathcal{P}'_u, \mathcal{I}'_u)$ and the maximal element $(\mathcal{P}\,'_u, \mathcal{I}\,'_u)$.

2. $\mathbb{L}_u(\mathcal{R})$ is the class of the factorization structures $(\mathcal{E}, \mathcal{M})$, for which $\mathcal{I}'_u \subset \mathcal{M} \subset \mathcal{I}\,'_u$.

**Lemma 3.** Let $m: X \to Y$ be an universal monomorphism. Then $\pi(m)$ is a sectional morphism.

**Theorem 4.** 1. Let $(\mathcal{E}, \mathcal{M}) \in \mathbb{L}_u(\mathcal{R})$. Then $r: \mathcal{C}_2\mathcal{V} \to \mathcal{R}$ is a $(\mathcal{E}, \mathcal{M})$-functor: $r(\mathcal{E}) \subset \mathcal{E}$ and $r(\mathcal{M}) \subset \mathcal{M}$.

2. $f \in \mathcal{P}\,'(\mathcal{R}) \iff r(f) \in \mathcal{P}\,'(\mathcal{R})$. 

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Keywords: Reflector functor, lattice

2010 AMS Classification: 55P65

References:

2-Absorbing $\delta$-Primary Fuzzy Ideals of Commutative Rings

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In this work, we define 2-Absorbing $\delta$-primary fuzzy ideals which is the generalizations of 2-absorbing fuzzy ideal and 2-absorbing primary fuzzy ideals. Furthermore, we give some fundamental results concerning these notions.

Keywords: 2-Absorbing $\delta$-Primary fuzzy ideals, 2-Absorbing primary fuzzy ideals.

2010 AMS Classification: 03E72

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Relation Between Hyper EQ-algebras and Some Other Hyper Structures

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In this study by considering the notion of hyper EQ-algebra, as a generalization of EQ-algebra (algebra of truth values for a higher-order fuzzy logic), we define some types of filters in this structure and investigated some related results. Then we find the relation between hyper EQ-algebras and hyper BCK-algebras, hyper MV-algebras and (weak) hyper residuated lattices.

\textbf{Keywords:} Hyper EQ-algebra, hyper BCK-algebra, hyper MV-algebra, (weak) hyper residuated lattice

\textbf{2010 AMS Classification:} 20N20

\textbf{References:}

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Fuzzy hyperideals in ordered semihyperrings

Osman KAZANCI¹, Şerife YILMAZ ², Bijan DAVVAZ³

In this study, we introduce the concept of fuzzy hyperideals of ordered semihyperrings, which is a generalization of the concept of fuzzy hyperideals of semihyperrings to ordered semihyperring theory. We investigate its related properties. We show that every fuzzy quasi-hyperideal is a fuzzy bi-hyperideal and in a regular ordered semihyperring, fuzzy quasi-hyperideal and fuzzy bi-hyperideal coincide.

Keywords: Semihyperring: ordered semihyperring: fuzzy hyperideal.

2010 AMS Classification: 03E72; 97H50.

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Fuzzy interior hyperideals in ordered semihyperrings

Şerife YILMAZ ¹, Osman KAZANCI ²

In this study, we introduce the concept of fuzzy interior hyperideals in ordered semihyperrings, which are a new sort of fuzzy hyperideals of semihyperrings. We investigate some of their related properties. We give a characterization of fuzzy interior hyperideals in terms of their level subsets. We show that every fuzzy hyperideal is a fuzzy interior hyperideal. We introduce the concept of intra-regular ordered semihyperrings and show that fuzzy hyperideals and fuzzy interior hyperideals coincide in an intra-regular ordered semihyperring. Finally, we introduce the concept of fuzzy simple ordered semihyperrings and prove some results.

Keywords: Ordered semihyperring: interior hyperideal: fuzzy interior hyperideal.

2010 AMS Classification: 03E72; 97H50.

Reference(s):
HYPERHILBERT SPACES
SAEID GHOLAMPOOR\textsuperscript{1} and ALI TAGHAVI\textsuperscript{1}

In this study we introduce the concept of hyper Hilbert spaces and prove some result such as, orthogonal projection and Riesz theorem about them.

Keywords: Hyperhilbert space, hilbert space, hyper space

2010 AMS Classification: 20N20

References:
3. F. Marty, Sur une generalization de la notion de group, 8th congress of the Scandinavian Mathematics, Stockholm, (1934), 45{49.
9. A. Taghavi and R. Hosseinzadeh and H. Rohi, Hyperinner product spaccs,

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The Lattice Structure of Subhypergroups of a Hypergroup

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In mathematics, determination of algebraic structures is very important. Many methods have been applied in determining these structures until today. One of them is investigating the lattice structure of substructures of algebraic structures (such as submonoids of a monoid, subgroups of a group, ideals of a ring, submodules of a module, subspaces of a vector space, etc.) according to the inclusion relation. As a generalization of algebraic structures, hyperstructure was defined in 1934 by F. Marty. Since then this theory has been developed by many mathematicians. In the last fifteen years, various applications of algebraic structures (in geometry, binary relations, lattices, fuzzy sets, rough sets, automata, cryptography, codes, median algebra, relational algebra, artificial intelligence probability) have been obtained.

In this study, we investigate the properties of closed, invertible, ultraclosed and conjugable subhypergroups classes. We study when the hypergroups satisfy the property that the hyperproduct of subhypergroups becomes an operation on the set of subhypergroups. It is investigated in which cases, the poset of the subhypergroups of a hypergroup is a lattice. It is examined when this lattice is modular or distributive. Thus some information about a hypergroup may be obtained by investigating the lattice of its subhypergroups.

**Keyword(s):** subhypergroups, lattice.

**2010 AMS Classification:** 20N20, 06B99.

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Intuitionistic Fuzzy Weakly Prime Ideals

Tuğba Arkan¹, Serkan Onar¹, Deniz Sönmez¹, Bayram Ali Ersoy¹

In this study, the fundamental definitions and theorems regarding intuitionistic fuzzy sets and intuitionistic fuzzy ideals of commutative ring with identity $R$ have been given as preliminaries. After the preliminaries, we introduce the notions of intuitionistic fuzzy weakly prime ideals, intuitionistic fuzzy partial weakly prime ideals, intuitionistic fuzzy weakly semiprime ideals of $R$. Let $P = \langle \mu_P, \nu_P \rangle$ be a nonconstant intuitionistic fuzzy ideal of $R$. If $(0,1) \neq A \cdot B \subseteq P$ implies $A \subseteq P$ or $A \subseteq P$ where $A = \langle \mu_A, \nu_A \rangle$, $B = \langle \mu_B, \nu_B \rangle$ intuitionistic fuzzy ideals of $R$, then $P$ is called intuitionistic fuzzy weakly prime ideal of $R$. If $P(xy) = P(x)$ or $P(xy) = P(y)$ for $xy \neq 0$, then $P$ is called intuitionistic fuzzy partial weakly prime ideal of $R$. A nonconstant intuitionistic fuzzy ideal $P$ is called intuitionistic fuzzy weakly semiprime ideal of $R$ if $(0,1) \neq B^2 \subseteq P$ implies $B \subseteq P$ where $B$ is an intuitionistic fuzzy ideal of $R$. Also, we give some relations between intuitionistic fuzzy weakly prime ideals and weakly prime ideals of $R$.

Keywords: Intuitionistic fuzzy prime ideals, intuitionistic fuzzy weakly prime ideals, intuitionistic fuzzy partial weakly prime ideals, intuitionistic fuzzy weakly semiprime ideals.

2010 AMS Classification: 03F55, 03E72, 08A72.

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Some Results on Tensor Product of Krasner Hypervector Spaces

R. Ameri¹, K. Ghadimi²

We introduce and study tensor product of Krasner hypervector spaces (Krasner hyperspaces). Here we introduce the (resp. multivalued) middle linear maps of Krasner hyperspaces and construct the categories of linear maps and multivalued linear maps of Krasner hyperspaces. It is shown the tensor product of two Krasner hyperspaces, as an initial object in this category, exists. Also, notion of a quasi-free object in category of Krasner hyperspaces are introduced and it is proved that in this category a quasi-free object up to maximum is unique.

Keyword(s): Krasner hypervector space, Multivalued middle linear map, Quasi-free, Tensor product

2010 AMS Classification: 20N20

1 Introduction

The theory of algebraic hyperstructures is a well-established branch of classical algebraic theory. Hyperstructure theory was first proposed in 1934 by Marty, who defined hypergroups and began to investigate their properties with applications to groups, rational fractions and algebraic functions [10]. It was later observed that the theory of hyperstructures has many applications in both pure and applied sciences; for example, semihypergroups are the simplest algebraic hyperstructures that possess the properties of closure and associativity. The theory of hyperstructures has been widely reviewed ([6], [7], [8], [9] and [12]) (for more see [1, 2, 3, 4, 5]).

In [11] M. Motameni et. al. studied hypermatrix. R. Ameri in [1] introduced and studied categories of hypermodules. Let $V$ and $W$ be two Krasner hyperspaces over the hyperfiled $K$. The purpose of this paper is the study of tensor product of Krasner hyperspaces. We introduce the category of multivalued linear maps of Krasner hyperspaces and then construct the tensor product of $V$ and $W$ as initial object in this category.

2 Preliminaries and main results

Let $H$ be a nonempty set. A map $\cdot : H \times H \rightarrow P^*(H)$ is called hyperoperation or join operation, where $P^*(H)$ is the set of all nonempty subsets of $H$. The join operation is extended to nonempty subsets of $H$ in natural way, so that $A \cdot B$ is given by

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\[ A \cdot B = \bigcup \{a \cdot b \mid a \in A \text{ and } b \in B \}. \]

the notations \( a \cdot A \) and \( A \cdot a \) are used for \( \{a\} \cdot A \) and \( A \cdot \{a\} \) respectively. Generally, the singleton \( \{a\} \) is identified by its element \( a \).

**Definition 1.** [7] A semihypergroup \((H, +)\) is called a canonical hypergroup if the following conditions are satisfied:

(i) \( x + y = y + x \) for all \( x, y \in R \);

(ii) There exists \( 0 \in R \) (unique) such that for every \( x \in R \), \( x \in 0 + x = x \);

(iii) For every \( x \in R \), there exists a unique element, say \( x' \) such that \( 0 \in x + x' \) (we denote \( x' = -x \));

(iv) For every \( x, y, z \in R \), \( z \in x + y \iff x \in z - y \iff y \in z - x \);

from the definition it can be easily verified that \( -(−x) = x \) and \( -(x + y) = −x − y \).

**Definition 2.** [7] Let \((K, +, ∗)\) be a hyperfield and \((V, +)\) be a canonical hypergroup. We define a Krasner hyperspace over \( K \) to be the quadruple \((V, +, ∙, K)\) where \( ∙ \) is a single-valued operation \( ∙ : K \times V \to V \) such that for all \( a \in K \) and \( x \in V \) we have \( a \cdot x \in V \), and for all \( a, b \in K \) and \( x, y \in V \) the following conditions are satisfied:

\[
\begin{align*}
(H_1) \quad a \cdot (x + y) &= a \cdot x + a \cdot y; \\
(H_2) \quad (a + b) \cdot x &= a \cdot x + b \cdot x; \\
(H_3) \quad a \cdot (b \cdot x) &= (a \cdot b) \cdot x; \\
(H_4) \quad 0 \cdot x &= 0; \\
(H_5) \quad 1 \cdot x &= x.
\end{align*}
\]

**Remark 1.** For simplicify, we say \( V \) is a \( Kr \)-hyperspace.
Definition 4. Let \((F, \cdot)\) be an object in the category \(Kr - Hvect\) and \(i : X \hookrightarrow F\) be an inclusion map of sets. We say that \(F\) is quasi-free on the subset \(X\) provided that:

(i) \(F = \langle X \rangle\);

(ii) For any object \(V\) in \(Kr - Hvect\) and any multivalued map \(\lambda : X \rightarrow P^*(V)\), there is a maximum \(smv, \overline{\lambda} : F \rightarrow P^*(V)\) such that for all \(x \in X\), we have \(\overline{\lambda}i(x) = \lambda(x)\).

Definition 5. Let \(V\) and \(W\) be two \(Kr\)-hyperspaces over a hyperfield \(K\). Let \(F\) be the free abelian group on the set \(V \times W\). Let \(H\) be the subgroup of \(F\) generated by all elements of the following forms (for all \(v, v' \in V, w, w' \in W\), and \(a \in K\)):

(i) \((v + v', w) - (v, w) - (v', w)\), where \((v + v', w) = \bigcup_{t \in v + v'} (t, w)\)

(ii) \((v, w + w') - (v, w) - (v, w')\);

(iii) \((a \cdot v, w) - (v, a \cdot w)\).

The quotient group \(F/H\) is called a tensor product of \(V\) and \(W\); it is denoted \(V \otimes_K W\). The coset \((v, w) + K\) of the element \((v, w)\) in \(F\) is denoted \(v \otimes w\); the coset of \((0, 0)\) is denoted 0.

Theorem 1. Let \(F\) be a \(Kr\)-hyperspace over a hyperfield \(K\) and \(X\) be a basis for \(F\). Then

(i) If \(j : X \hookrightarrow F\) is a inclusion map, then for all \(Kr\)-hyperspace \(V\) and map \(f : X \rightarrow P^*(V)\), there is a maximum \(smv, \varphi : F \rightarrow P^*(V)\) such that the diagram

\[
\begin{array}{ccc}
X & \xrightarrow{j} & F \\
\downarrow f & & \downarrow \varphi \\
P^*(V) & & 
\end{array}
\]

is commutative.

(ii) For all \(Kr\)-hyperspace \(V\) and \(f : X \rightarrow P^*(V)\) induced maximum \(smv, \varphi : F \rightarrow P^*(V)\), means there is a maximum \(smv, \varphi : F \rightarrow P^*(V)\) such that \(\varphi|_X = f\).
Given hyperspaces $V$ and $W$ over a hyperfield $K$, it is easy to verify that the map $i : V \times W \to V \otimes_K W$ given by $(v, w) \mapsto v \otimes w$ is a middle linear map. The map $i$ is called the canonical middle linear map. Its importance is seen in

**Theorem 2.** Let $V$ and $W$ be $Kr$-hyperspaces over a hyperfield $K$, and let $Z$ be an abelian group. If $g : V \times W \to Z$ is a middle linear map, then there exists unique group homomorphism $\bar{g} : V \otimes_K W \to Z$ such that $\bar{g}i = g$, where $i : V \times W \to V \otimes_K W$ is the canonical middle linear map. $V \otimes_K W$ is uniquely determined up to isomorphism by this property. In other words $i : V \times W \to V \otimes_K W$ is universal in the category $\mathcal{ML}(V, W)$ of all middle linear maps on $V \times W$.

**Reference(s):**

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Fuzzy coprimary submodules and their representation

Jafar A'zami

Let R be a commutative ring with non-zero identity and let M be a non-zero unitary R-module. The concept of fuzzy coprimary submodule as a dual notion of fuzzy primary will be studied. Among other things, the behavior of this concept with respect to fuzzy localization formation and fuzzy quotient will be examined. Also the uniqueness theorem for a non-zero fuzzy representable submodule of M will be proved.

Keywords: Fuzzy coprimary submodule, Fuzzy prime and primary ideal, Fuzzy localization, fuzzy coprimary representation, fuzzy attached primes.

2010 AMS Classification: 08A72

Reference(s):
Constructing Topological Hyperspace with Soft Sets

Güzide Şenel¹

In this study, by defining soft ditopological spaces, I construct a topological hyperspace with soft sets. I make a new approach to the soft topology via soft set theory, with defining two structures on a soft set - a soft topology and a soft subspace topology. Moreover, I characterize separation axioms in soft ditopological spaces and investigate the relations between soft topological and soft ditopological structures [10]. Based on this idea, the relations between the separation axioms of ordered soft topological spaces and the separation axioms of the corresponding soft ditopological spaces are established.

Keywords: Soft sets, hyperspace, topological hyperspace

2010 AMS Classification: 54B20

References:

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Fuzzy Weakly Prime $\Gamma$-Ideal in $\Gamma$-Rings

Gülşah Yeşilkurt$^1$, Serkan Onar$^1$, Deniz Sönmez$^1$ and Bayram Ali Ersoy$^1$

In this study, we investigate fuzzy weakly prime, fuzzy partial weakly prime and fuzzy weakly semiprime $\Gamma$-ideal of a $\Gamma$-ring. We obtain some characterizations of fuzzy weakly prime, fuzzy partial weakly and fuzzy weakly semiprime $\Gamma$-ideal of a $\Gamma$-ring. First we give the definition of fuzzy weakly prime ideal, fuzzy weakly semiprime and fuzzy partial weakly prime $\Gamma$-ideal. Further we give some properties of its.

**Keywords:** Fuzzy prime ideal, fuzzy weakly prime $\Gamma$-ideal, fuzzy partial weakly prime $\Gamma$-ideal, fuzzy weakly semiprime $\Gamma$-ideal.

**2010 AMS Classification:** 03E72, 16D25, 16P99, 08A72.

**References:**


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Intuitionistic Fuzzy 2-Absorbing Ideals of Commutative Rings

Sanem YAVUZ, Serkan ONAR, Deniz SONMEZ, Bayram Ali ERSOY

The aim of this paper is to give a definitions of intuitionistic fuzzy 2- absorbing ideals and intuitionistic fuzzy weakly completely 2- absorbing ideals of commutative rings and to give their properties. Moreover, we give diagram which transition between definitions of intuitionistic fuzzy 2- absorbing ideals of commutative rings.


2010 AMS Classification: 08A72

References:

Transition from Two-Person Zero-Sum Games to Cooperative Games with Fuzzy Payoffs

Adem C. Cevikel \(^1\) and Mehmet Ahlatcioğlu \(^2\)

In this study, we deal with games with fuzzy payoffs. We proved that players who are playing a zero-sum game with fuzzy payoffs against nature are able to increase their joint payoff, and hence their individual payoffs by cooperating. It is shown that, a cooperative game with the fuzzy characteristic function can be constructed via the optimal game values of the zero-sum games with fuzzy payoffs against nature at which players' combine their strategies and act like a single player. It is also proven that, the fuzzy characteristic function that is constructed in this way satisfies the superadditivity condition. Thus we considered a transition from two-person zero-sum games with fuzzy payoffs to cooperative games with fuzzy payoffs. The fair allocation of the maximum payoff (game value) of this cooperative game among players is done using the Shapley vector.

**Keywords:** Cooperative game; Fuzzy number; Fuzzy games; Shapley vector.

**2010 AMS Classification:** 91A10, 91A12, 03E72.

**References:**
1. L.A.Zadeh, Fuzzy sets, Information and control, 8, (1965), 338-353.

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On computation of fundamental group of a finite hypergroup

K. Abbasi¹, R. Ameri², Y. Talebi-Rostami³

The purpose of this paper is the computation of fundamental group in a finite hypergroup \((H, \circ)\). In this regards we first obtain an algorithm to construct the equivalence classes of \(\beta\), the fundamental relation on \(H\), then we construct \((H/\beta, \otimes)\), the fundamental group of \(H\). In particular, given some classes of hypergroups, we find fundamental groups of them. We apply a comprehensive Java program to compute the fundamental group of a given finite hypergroup. It consists of two sub-programs (Hypergroup generator and Main). Hypergroup generator, counts all hypergroups of order \(n \leq 3\) \((n \in \mathbb{N})\) and isomorphism classes of them and enumerates quasihypergroups of order \(n\) and all \(\beta\)-equivalence classes and by the next sub-program (Main), it is checked that is an arbitrary hypergroupoid \((H, \circ)\) of order \(n \ (n \in \mathbb{N})\) is a hypergroup or not. If it is a hypergroup, this sub-program computes its \(\beta\)-equivalence classes and fundamental group.

Keywords: Hypergroup, Fundamental relation, Fundamental group, Computation.


Acknowledgements:

The author partially has been supported by "Algebraic Hyperstructure Excellence (AHETM), Tarbiat Modares University, Tehran, Iran" and "Research Center in Algebraic hyperstructures and Fuzzy Mathematics, University of Mazandaran, Babolsar, Iran".

References:


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Various Kinds of Quotient of a Canonical Hypergroup
Hossein Shojaei$^1$ and Reza Ameri$^1$

In this study various kinds of quotients of a given canonical hypergroup are introduced and studied. In this regards we introduce some regular equivalence relations on a canonical hypergroup to construct different quotients for this hyperstructure. We will proceed to investigate the relationships among these relations such that these extracted quotient structures be equal. Also, the relationship between the heart of a given canonical hypergroup and its quotient via an equivalence relation is studied and some related basic results are obtained. Finally, we study the quotient hyperstructures of a canonical hypergroup induced via a normal canonical subhypergroup, and show that for this special kind of quotient space all various kinds of quotients are concid.

Keywords: Hypergroup, Canonical Hypergroup, Quotient Hypergroup, Heart.

2010 AMS Classification: 20N20, 20N15

References:


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On multipliers of hyper BCC-algebras

Didem SÜRGEVİL UZAY\textsuperscript{1}, Alev FIRAT\textsuperscript{2}

In this paper, we introduced the notion of multiplier of a hyper BCC-algebra, and investigated some properties of hyper BCC-algebras. And then we introduced notion of kernels. Also we gave some propositions related with isotone and $\text{Fix}_d(H)$.

\textbf{Keywords:} hyper BCC-algebra, multiplier, isotone, $\text{Fix}_d(H)$, regular.

\textbf{2010 AMS Classification:} 20N20, 16W25

\textbf{Reference(s):}

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Derivations on Hyperlattices

Şule Ayar ÖZBAL¹, Alev FIRAT²

In this paper, we introduced the notion of derivations on hyperlattices and investigated some related properties. Also, we characterized the Fix₀(L), and Ker₀(L) by derivations

Keyword(s): Lattices, hyperlattices, derivations.

2010 AMS Classification: 03G10, 20N20, 16W25

References:
On fuzzy $\phi$-prime ideals

Naser Zamani

Let $R$ be a commutative ring with identity. Let $FI(R)$ be the set of all fuzzy ideals of $R$ and $\phi : FI(R) \rightarrow FI(R) \cup \{0_R\}$ be a function. We introduce the concept of fuzzy $\phi$-prime ideals. Some relationships between fuzzy $\phi$-prime ideals and prime ideals of $R$ will be investigated. We find conditions under which fuzzy $\phi$-primness gives primness and vice versa. Also, the behaviour of this concept in rings product will be studied.

Keywords: Fuzzy $\phi$-prime ideals, fuzzy prime ideal

2010 AMS Classification: 08A72

References:
On pure hyperideals in ordered semihypergroups

Thawhat Changphas\(^1\) and Bijan Davvaz\(^2\)

In this study, the notions of pure hyperideal, weakly pure hyperideal and purely prime hyperideal in ordered semihypergroups are introduced and studied. We prove that the set of all purely prime hyperideals is topologized.

**Keywords:** Algebraic hyperstructure, ordered semihypergroup, weakly regular, pure hyperideal, weakly pure hyperideal, purely prime hyperideal, topology

**2010 AMS Classification:** 20N20

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Relation Between Hyper \( K \)-algebras and Superlattice (Hypersemilattice)

A. Rezazadeh\(^1\), A. Radfar\(^2\), R. A. Borzooei\(^3\)

In this paper, by considering the notions of hypersemilattice and superlattice, we prove that any commutative and positive imolicative hyper \( K \)-algebra, is a hypersemilattice. Moreover, we prove that any bounded commutative hyper \( K \)-algebra with condition \( L \) is a superlattice.

1. Introduction

The theory of hyperstructures was introduced in 1934 by Marty [5]. This theory has been subsequently developed by the contribution of various authors. In [1], R. A. Borzooei et al. applied the hyperstructures to \( K \)-algebras and introduced the notion of a hyper \( K \)-algebra and investigated some related properties. Some researchers applied the hyperstructure to some accepts of lattice theory and the notion of hypersemilattice was introduced by Z. Bin et al. in [2] and the notion of superlattice was introduced by Mittas and Konstantinidou in [6]. In this paper, we prove that every hyper \( K \)-algebra by some condition is a hypersemilattice. In follow, we introduce the notions \( \land \) and \( \lor \) on hyper \( K \)-algebras and we prove that every hyper \( K \)-algebra of order 3 by some condition is a superlattice.

2. Preliminary

In this section, we give some definitions and theorems that we need in the next sections.

**Definition 2.1.** [2] Let \( L \) be a nonempty set with a binary hyperoperation \( \otimes \) on \( L \) such that for all \( a, b, c \in L \), the following condition hold:
(i) \( a \in a \otimes a \),
(ii) \( a \otimes b = b \otimes a \),
(iii) \( (a \otimes b) \otimes c = a \otimes (b \otimes c) \).

Then \( (L, \otimes) \) is called a hypersemilattice.

**Definition 2.2.** [6] A superlattice is a partially ordered set \( (S, <) \) with two hyper operations \( \lor \) and \( \land \) such that the following properties hold:
(S1) \( a \in a \lor a \) and \( a \in a \land a \),
(S2) \( a \lor b = b \lor a \) and \( a \land b = b \land a \).

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(S3) \((a \cup b) \cup c = a \cup (b \cup c)\) and \((a \cap b) \cap c = a \cap (b \cap c)\),
(S4) \(a \in a \cup (a \cap b)\) and \(a \in a \cap (a \cup b)\),
(S5) if \(a < b\) then \((b \in a \cup b\) and \(a \in a \cap b)\),
(S6) \((b \in a \cup b\) or \(a \in a \cup b)\) implies \(a < b\).
for all \(a, b, c \in S\)

**Definition 2.3.** [1] By a hyper \(K\)-algebra we mean a nonempty set \(H\) endowed with a hyperoperation "\(\circ\)" and a constant 0 satisfy the following axioms:

(HK1) \((x \circ z) \circ (y \circ z) < x \circ y)\),
(HK2) \((x \circ y) \circ z = (x \circ z) \circ y)\),
(HK3) \(x < x)\),
(HK4) \(x < y\) and \(y < x)\) imply \(x = y)\),
(HK5) \(0 < x)\).
for all \(x, y, z \in H)\), where \(x < y\) is defined by \(0 \in x \circ y\) and for every \(A, B \subseteq H\), \(A < B\) is defined by \(\exists a \in A, \exists b \in B\) such that \(a < b\).

**Theorem 2.4.** [1] Let \(H\) be a hyper \(K\)-algebra. Then the following are hold:

\((i)\) \(x \in x \circ 0\),
\((ii)\) \(x \circ y < z) \iff x \circ z < y\),
\((iii)\) \(x \circ (x \circ y) < y)\),
\((iv)\) \(x \circ y < x)\),
\((v)\) \(A \circ B < A\).
for all \(x, y, z \in H\).

3. Relation between hyper \(K\)-algebras and hypersemilattice
In this section we prove that every commutative and positive implicative hyper \(K\)-algebra is a hypersemilattice.

**Definition 3.1.** A hyper \(K\)-algebra \((H, \circ, 0)\) is called commutative if for all \(x, y \in H)\),
\(x \circ (x \circ y) = y \circ (y \circ x)\)

**Notation.** In any commutative hyper \(K\)-algebra, for all \(x, y \in H\), we denote
\(x \cap y = \{z \mid z \in y \circ (y \circ x)\}\)

**Theorem 3.2.** Let \(H\) be a commutative hyper \(K\)-algebra. Then we have the following properties:

\((i)\) \(x \cap y < x \) and \(x \cap y < y)\),
\((ii)\) \(x \cap y = y \cap x)\),

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(iii) \( x \in x \cap x \),
(iv) If \( x < y \), then \( x \in x \cap y \).
for all \( x, y \in H \).

**Definition 3.3.** A hyper \( K \)-algebra \((H, \circ)\) is said to be positive implicative if for all \( x, y, z \in H \),
\[
(x \circ y) \circ z = (x \circ z) \circ (y \circ z)
\]

**Theorem 3.4.** Let \( H \) be a commutative and positive implicative hyper \( K \)-algebra. Then for all \( x, y, z \in H \),
\[
(x \cap y) \cap z = x \cap (y \cap z)
\]

**Corollary 3.5.** Let \((H, \circ)\) be a commutative and positive implicative hyper \( K \)-algebra. Then \((H, \cap)\) is a hypersemilattice.

**Example 3.6.** Let \( H = \{0, a, b\} \) and the hyper operation \( \circ \) is defined on \( H \) as follows:

\[
\begin{array}{ccc}
| & 0 & a & b \\
\hline
0 & \{0\} & \{0, a\} & \{0, a, b\} \\
a & \{a\} & \{0, a\} & \{0, a, b\} \\
b & \{b\} & \{b\} & \{0, a, b\}
\end{array}
\]

Then \((H, \circ)\) is a commutative and positive implicative hyper \( K \)-algebra. Also we can see that \((H, \cap)\) is a hypersemilattice.

4. Relation between hyper \( K \)-algebras and superlattice

In this section we introduce the notion hypermeet \( \wedge \) on hyper \( K \)-algebras.

**Definition 4.1.** A hyper \( K \)-algebra \((H, \circ, 0)\) is called to be bounded, if there exist an element 1 such that \( x < 1 \), for all \( x \in H \) and is called complemented, if \( H \) is bounded and \( 1 \circ x \) has a least element with respect to \( < \), for all \( x \in H \).

We note that if \( H \) is bounded, then by \((HK4)\) we can easily get that 1 is unique. Also if \( H \) be complemented, then we use \( x' \) to denote \( \text{min}(1 \circ x) \).

**Notation:** In any commutative hyper \( K \)-algebra, we denote
\[
x \Lambda y = \{z \mid z \in y \circ (y \circ x) \text{ s.t } z < x \text{ and } z < y\},
\]
for all \( x, y \in H \).

**Theorem 4.2.** Let \( H \) be a commutative hyper \( K \)-algebra. If \((x \Lambda y) \Lambda z = x \Lambda (y \Lambda z)\), for all \( x, y, z \in X \), then \( x \Lambda y \) is a greatest lower bound of \( x \) and \( y \).

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Definition 4.3. Let $H$ be a bounded commutative complemented hyper $K$-algebra. We say $H$ satisfies in conditions $L$, if for all $x, y \in H$,

(L1) $x \land y \neq \emptyset$,
(L2) $x' \circ y = y' \circ x$,
(L3) $(x')' = x$.

Notation. In any bounded commutative hyper $K$-algebra with condition $L$, we define

$$x \lor y = \{z \mid z \in (x' \land y')'\}$$

for all $x, y \in H$.

Proposition 4.4. Let $H$ be a bounded commutative hyper $K$-algebra with condition $L$. Then the following hold:

(i) $x < x \lor y$ and $y < x \lor y$,
(ii) $x \in x \lor x$,
(iii) $x \lor y = y \lor x$,
(iv) If $x < y$, then $y \in x \lor y$,
(v) If $(x \in x \land y$ or $y \in x \lor y)$, then $x < y$,
for all $x, y \in H$.

Theorem 4.5. Let $H$ be a bounded commutative hyper $K$-algebra with condition $L$. If $(x \land y) \land z = x \land (y \land z)$, then for all $x, y, z \in H$, $(x \lor y) \lor z = x \lor (y \lor z)$.

Theorem 4.6. Let $H$ be a bounded commutative hyper $K$-algebra with condition $L$ and $\land$ be associative. Then $x \lor y$ is a lowest upper bound of $x$ and $y$, for all $x, y, z \in H$.

Theorem 4.7. Let $H$ be a bounded commutative hyper $K$-algebra with condition $L$. Then for any $x, y \in H$, we have $x \in x \land (x \lor y)$ and $x \in x \lor (x \land y)$.

Corollary 4.8. Let $H$ be a bounded commutative hyper $K$-algebra with condition $L$ and $\land$ be associative. Then $(H, \land, \lor)$ is a superlattice.

5. Bounded commutative hyper $K$-algebra of order 3 with condition $L$

In this section we prove that every hyper $K$-algebra of order 3 with condition $L$ is a superlattice.

Theorem 5.1. Let $H = \{0, a, 1\}$ be a bounded commutative hyper $K$-algebra with condition $L$. Then $(x \land y) \land z = x \land (y \land z)$, for all $x, y, z \in H$.

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Corollary 5.2. Let $H = \{0, a, 1\}$ be a bounded commutative hyper $K$-algebra with condition $L$. Then $(H, \land, \lor)$ is a superlattice.

In the next example we show that Theorem 5.1 is not correct for a hyper $K$-algebra of order more than 3 in general.

Example 5.3. Let $H = \{0, a, b, 1\}$ and the hyper operation "$\circ$" defined as follows:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>a</th>
<th>b</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>{0, a, b, 1}</td>
<td>{0, a, b, 1}</td>
<td>{0}</td>
<td>{0}</td>
</tr>
<tr>
<td>a</td>
<td>{a,b,1}</td>
<td>{0, a, b, 1}</td>
<td>{0,a}</td>
<td>{0}</td>
</tr>
<tr>
<td>b</td>
<td>{b}</td>
<td>{b}</td>
<td>{0, a, b, 1}</td>
<td>{0, a, b, 1}</td>
</tr>
<tr>
<td>1</td>
<td>{1}</td>
<td>{b}</td>
<td>{a,b,1}</td>
<td>{0, a, b, 1}</td>
</tr>
</tbody>
</table>

Then $(H, \circ)$ is a bounded commutative hyper $K$-algebra and satisfies in condition $L$. We can see that $\land$ is not associative operator.

Keyword(s): Hyper $K$-algebra, hypersemilattice, superlattice.

2010 AMS Classification: 03G10,06F35

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VAGUE SOFT HYPERMODULES

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In this study, the notion of vague soft hypermodules as an extension of the notion of vague soft hypergroups and vague soft hyperrings is introduced. Then some basic properties of vague soft sets and homomorphisms between vague soft hypermodules are presented. Also we studied the image and inverse image of a vague soft hypermodule under a vague soft hypermodule homomorphism.

Keywords: Soft set, vague soft set, vague soft hypermodule.

2010 AMS Classification: 06D72, 08A99, 20N20, 08A72.

References:


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An introduction to Zero-Divisor Graph of a Commutative Multiplicative Hyperring

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The purpose of this note is the study of zero-divisor graph of a commutative multiplicative hyperring, as a generalization of commutative rings. In this regards we consider a commutative multiplicative hyperring \((R, +, \circ)\), where \((R, +)\) is an abelian group, \((R, \circ)\) is a semihypergroup and for all \(a, b, c \in R\), \(a \circ (b + c) \subseteq (a \circ b) + (a \circ c)\) and \((a + b) \circ c \subseteq (a \circ c) + (b \circ c)\). For \(a \in R\) a non-zero element \(b \in R\) is said to be a zero-divisor of \(a\), if \(0 \in a \circ b\).

The set of zero-divisors of \(R\) is denoted by \(Z(R)\). We associative to \(R\) a zero-divisor graph \(\Gamma(R)\), whose vertices of \(\Gamma(R)\) are the elements of \(Z(R)^* = Z(R)\setminus\{0\}\) and two distinct vertices of \(\Gamma(R)\) are adjacent if they were in \(Z(R)\). Finally, we obtain some properties of \(\Gamma(R)\) and compare some of its properties to the zero-divisor graph of a classical commutative ring and show that almost all properties of zero-divisor graphs of a commutative ring can be extend to \(\Gamma(R)\) while \(R\) is a strongly distributive multiplicative hyperring.

**Keywords:** Multiplicative hyperring, Zero-divisor graph, Fundamental relation.

**AMS 2010 Classification:** 20N20

**References:**


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Some ordered hypersemigroups which enter their properties into their σ-classes

Niovi Kehayopulu

We are interested in ordered hypersemigroups $H$ which enter their properties into their σ-classes, where σ is a complete semilattice congruence on $H$. This gives information about the structure of ordered hypersemigroups referring to the decomposition of these hypersemigroups into components of the same type. We prove the following:

**Theorem 1.** If $H$ is a regular, left (resp. right) regular or intra-regular ordered hypersemigroup and σ a complete semilattice congruence on $H$, then the σ-class $(a)_σ$ of $H$ is, respectively, a regular, left (resp. right) regular or intra-regular (ordered) subsemigroup of $H$ for every $a \in H$.

As a consequence, if $H$ is a completely regular ordered hypersemigroup and σ a complete semilattice congruence on $H$, then the σ-class $(a)_σ$ is a completely regular subsemigroup of $H$ for every $a \in H$.

**Theorem 2.** If $H$ is a left (resp. right) quasi-regular or semisimple ordered hypersemigroup and σ a complete semilattice congruence on $H$, then the σ-class $(a)_σ$ of $H$ is, respectively so.

**Theorem 3.** If $H$ is a left (resp. right) simple ordered hypersemigroup and σ a complete semilattice congruence on $H$, then $(a)_σ$ is a left (resp. right) simple subsemigroup of $H$ for every $a \in H$.

**Theorem 4.** If $H$ is a simple ordered hypersemigroup and σ a complete semilattice congruence on $H$, then $(a)_σ$ is a simple subsemigroup of $H$ for every $a \in H$.

**Theorem 5.** If $H$ is an archimedean or weakly commutative ordered hypersemigroup and σ a complete semilattice congruence on $H$, then $(a)_σ$ is, respectively so.

The "⇐-part" of the theorems above being obvious, we get characterizations of the above mentioned types of ordered hypersemigroups via their σ-classes, σ being a complete semilattice congruence on $H$.

**Keywords:** ordered hypersemigroup, regular, left regular, intra-regular, left quasi-regular, semisimple, left simple, simple, archimedean, weakly commutative, complete semilattice congruence.

**2010 AMS Classification:** 20M99 (06F05)
Study of $\Gamma$-hyperrings by fuzzy hyperideals with respect to a $t$-norm

Krisanthi Naka$^1$, Kostaq Hila$^2$, Serkan Onar$^3$, Bayram Ali Ersoy$^4$

In this paper, we inquire further into the properties on some kind fuzzy hyperideals and we study the $\Gamma$-hyperrings via $T$-fuzzy hyperideals. By means of the use of a triangular norm $T$, we define, characterize and study the $T$-fuzzy left and right hyperideals, $T$-fuzzy quasi-hyperideal and bi-hyperideal in $\Gamma$-hyperrings and some related properties are investigated. We compare fuzzy hyperideal to $T$-fuzzy hyperideals. We have shown that $\Gamma$-hyperring is regular if and only if intersection of any $T$-fuzzy right hyperideal with $T$-fuzzy left hyperideal is equal to its product. We introduce the notion of $T$-fuzzy quasi-hyperideal and $T$-fuzzy bi-hyperideal. We discuss some of its properties. We have shown that the meet of $T$-fuzzy right and $T$-fuzzy left ideal is a $T$-fuzzy quasi hyperideal of a $\Gamma$-hyperring. We characterize regular $\Gamma$-hyperring with $T$-fuzzy quasi-hyperideal and $T$-fuzzy bi-hyperideal. We also introduce the $T$-$(\lambda, \mu)$-fuzzy bi-hyperideals in $\Gamma$-hyperrings and investigate some of their properties.

Keyword(s): $\Gamma$-hyperrings, $t$-norm, $T$-fuzzy (resp. left, right) hyperideal, $T$-fuzzy quasi(bi)-hyperideal, $T$-$(\lambda, \mu)$-fuzzy bi-hyperideal.


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On an algebra of fuzzy $m$-ary semihypergroups

Krisanithi Naka¹, Kostaq Hila², Serkan Onar³, Bayram Ali Ersoy⁴

In this paper we deal with the fuzzy $m$-ary semihypergroups, fuzzy hyperideals and homomorphism theorems on $m$-ary semihypergroups and fuzzy $m$-ary semihypergroups. We also, introduce and study some classes of fuzzy hyperideals that of pure fuzzy, weakly pure fuzzy hyperideals in $m$-ary semihypergroups and some properties of them are investigated. We identify those $m$-ary semihypergroups for which every fuzzy hyperideal is idempotent. We also characterize the $m$-ary semihypergroups for which every fuzzy hyperideal is weakly pure fuzzy.

**Keywords:** $m$-ary semihypergroup, pure (weakly pure) fuzzy hyperideal, regular (weakly regular) $m$-ary semihypergroups.

**2010 AMS Classification:** 20N15, 03E72, 20N20.

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On Annihilator in Pseudo $BCI$-algebras

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In this paper, the concept of annihilator in a pseudo $BCI$-algebra is introduced and some related properties are investigated. Some necessary and sufficient conditions for a pseudo $BCI$-algebra to be semisimple are given. Moreover, it is proved that the annihilator of a closed ideal $A$, denoted by $A^*$, is the greatest closed pseudo $BCI$-ideal of $X$ contained in the $BCK$-part of $X$ and satisfied $A \cap A^* = \{0\}$.

**Keywords:** pseudo $BCI$-algebra, pseudo $BCI$-ideal, annihilator, normal ideal

**2010 AMS Classification:** 08A99, 03B60

**Acknowledgement:**

Authors thank the Research Council of Shahid Chamran University of Ahvaz for its financial support.

**References:**
The embedding of an ordered semihypergroup in terms of fuzzy sets

Krisanthi Naka¹, Kostaq Hila², Serkan Onar³, Bayram Ali Ersoy⁴

In this paper we have investigated an embedding theorem of ordered semihypergroups in terms of fuzzy sets. We prove that an ordered semihypergroup \( R \) is embedded in the set \( F(R) \) of all fuzzy subsets of \( R \), which is an \textit{poe}-semigroup with the ordered relation and the multiplication and addition defined in this paper.

Keywords: semihypergroup, ordered semihypergroup, fuzzy sets

2010 AMS Classification: 08A72, 20N20, 20N25, 06F05.

References:


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On $\delta$-Primary Hyperideals of Commutative Semihyperrings

Ashraf Abumghaiseeb $^1$ and Bayram Ali Ersoy $^1$

In this work, we study the mapping $\delta$ that assigns to each hyperideal $I$ of the commutative semihyperring $R$, another hyperideal $\delta(I)$ of the same semihyperring. Also we introduced the notation of $\delta$-zero divisor of commutative semihyperring and $\delta$-semidomainlike semihyperring which is generalization to those in the semirings. Moreover we showed that if $\delta$ be a global hyperideal expansion then $I$ is $\delta$-primary if and only if $Z_\delta(R/I) \subseteq \delta\left(\{0_{R/I}\}\right)$.

Keywords: Semihyperring, hyperideal, $\delta$-primary, $\delta$-semidomainlike semihyperring.

2010 AMS Classification: 20N99, 13A15

References:


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On the Vahlen Matrices

Mutlu Akar

After we recall definition of Vahlen Matrices, give their some properties.

Keyword(s): Clifford Algebra, Clifford Matrix, Möbius Transformations.


References:

On the Baireness of function spaces

Abderrahmane Bouchair

Let X be a topological space and $\alpha$ a nonempty family of compact subsets of X. Let $C_\alpha(X)$ denote the space of continuous real-valued functions on X equipped with the set open topology. A subfamily $\beta$ of $\alpha$ is called moving off $\alpha$ if, for each $A \in \alpha$ there is $B \in \beta$ with $A \cap B = \emptyset$. We say that X has the Moving Off Property (MOP) with respect to $\alpha$, if every subfamily of $\alpha$ which moves off $\alpha$ contains an infinite subfamily which has a discrete open expansion in X.

Recently, Bouchair and Kelaiaia [1] proved that, for X paracompact q-space then $C_\alpha(X)$ is Baire if and only if each point of X has a neighborhood from $\alpha$. In this work we prove that if X is first countable, then $C_\alpha(X)$ is a Baire space if and only if X has the Moving Off Property with respect to $\alpha$.

Keyword(s): Baire space, function space, set open topology, topological game

2010 AMS Classification: 54C35

Reference(s):
Global asymptotic stability of a higher order difference equation

Farida Belhannache¹, Nouressadat Touafek²

In this work, we investigate the global behavior of positive solutions of the difference equation

\[ x_{n+1} = \frac{A + Bx_{n-2k-1}}{C + D \prod_{i=l}^{k} x_{n-2i}}, \quad n = 0, 1, \ldots \]

with non-negative initial conditions, the parameters \(A, B\) are non-negative real numbers, \(C, D\) are positive real numbers, \(k, l\) are non-negative fixed integers and \(m_i, i \in \{l, \ldots, k\}\) are positive fixed integers such that \(l \leq k\).

**Keyword(s):** Difference equation, global behavior, oscillatory, boundedness

**2010 AMS Classification:** 39A10

**Reference(s):**

Weak $\alpha$-favorability of $C(X)$ with a set open topology

Kelaiaia Smail$^1$ and Harkat Lamia$^2$

Let $C(X)$ be the set of all continuous real valued functions we endow it with a set open topology with the help of a family of $\mathbb{R}$-compact subsets of $X$. We use a topological game to study the weak $\alpha$-favorability of $C(X)$.

**Keywords:** Function spaces, set-open topologies, uniform topologies, topological games, $\alpha$-favorability, $\mathbb{R}$-compact.

**2010 AMS Classification:** 54C35.

**References:**


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Curves on Lightlike Cone in Minkowski Space

Nemat Abazari\textsuperscript{1}, Alireza Sedaghatdoost\textsuperscript{2}

In the Minkowski space $\mathbb{E}^n_1$, the set of all lightlike vectors is called lightlike cone and it is denoted by $Q^{n-1}$. In this paper, we use Frenet orthonormal frame and asymptotic orthonormal frame for study of curves on the lightlike cone $Q^{n-1}$ in Minkowski space $\mathbb{E}^n_1$. We study all lightlike and spacelike curves in $Q^{n-1}$. We classify all curves with constant cone curvature in $Q^4$, $Q^5$ and $Q^6$. Also, we give some relation between Frenet curvature and cone curvature functions for a curve in $Q^3$.

\textbf{Keyword(s):} Asymptotic frame, cone curvature, lightlike cone, spacelike curve.

\textbf{2010 AMS Classification:} 53A35

\textbf{Reference(s):}
A class of LCD codes from group rings

Mehmet E. Koroglu¹, Bayram A. Ersoy¹

Linear codes with complementary duals (abbreviated LCD) are linear codes that they trivially intersect with their duals [1]. Finding new LCD code families are of great importance due to their wide range of applications. Unit derived group ring codes were given by Hurley and Hurley in [2]. Group rings are a rich source of unit elements. So it is possible to obtain many new code parameters. In this work, we provide a necessary and sufficient condition for the unit derived group ring codes to be LCD.

Keywords: Linear codes, LCD codes, group rings

2010 AMS Classification: 94B05, 94B60, 08A99

References:
New Blocking Cryptography Models

Sümeyra UÇAR¹, Nihal TAŞ², Nihal Yılmaz Özgür³

In this talk we present two new coding and decoding methods using Fibonacci $Q$ matrices and $R$-matrices. Since our methods study with small numbers, we obtain quite easy methods than the known methods in literature.

**Keyword(s):** Coding algorithm / decoding algorithm / Fibonacci $Q$ matrix / $R$ matrix

**2010 AMS Classification:** 68P30, 11B39, 1B37.

**Reference(s):**
A New Coding Theory with Generalized Pell \((p,i)\) – Numbers

Nihal Taş\(^1\), Sümeyra Uçar\(^2\), Nihal Yılmaz Özgür\(^3\)

Recently, it has been introduced a new coding algorithm, called \textit{blocking algorithm}, using Fibonacci (resp. Lucas) numbers and a blocking method. In this study, we develop a new coding and decoding method using the generalized Pell \((p,i)\) – numbers. We give an application of generalized Pell \((p,i)\) – numbers to \textit{blocking algorithm}.

\textbf{Keyword(s):} Coding theory / decoding theory / generalized Pell \((p,i)\) – numbers / blocking algorithm


\textbf{Reference(s):}

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Some Problems in Spectral Graph Theory
Sezer Sorgun¹, Hakan Küçük², Hatice Topcu³

Spectral Graph Theory is the studies of Linear Algebra and Graph Theory mix together. In this study, we usually find the relations between the eigenvalues of popular matrices and the graph parameters. In this talk, we present some problems related to energy problems, isomorphism etc. in the theory.

Keyword(s): Graph, Isomorphism, Energy, Graph Matrices, Eigenvalue

2010 AMS Classification: 05C50

Reference(s):
On Trees Which Have Four Non-Zero Randić Eigenvalues

Hakan Küçük¹, Sezer Sorgun², Hatice Topcu³

A popular and important research field is to investigate the characterization of the connected graphs with special and distinct eigenvalues. It is an interplay between combinatorics and linear algebra. Moreover, Randić Matrix and Randić Energy studies in Spectral Graph Theory are essential. In this presentation we give some basic information about Randić Matrix, then we present our observations and conclusions about trees which have four non-zero Randić eigenvalues.

Keyword(s): Graph, Matrices, Randić Matrix, Randić Eigenvalues, Trees

2010 AMS Classification: 05C50

Reference(s):
One-Parameter Planar Motions in Generalized Complex Number Plane $\mathbb{C}_p$

Nurten (Bayrak) Gürses$^1$ and Salim Yüce$^1$

The generalized complex numbers have the form

$$z = x + Jy, \ (x, y \in \mathbb{R}) \text{ where } J^2 = iq + p, \ (p, q \in \mathbb{R}).$$

By taking $J^2 = p; q = 0$ and $-\infty < p < \infty$, generalized complex number system can be presented as follows:

$$\mathbb{C}_p = \{x + Jy : x, y \in \mathbb{R}, J^2 = p\}.$$  

$\mathbb{C}_p$ is called $p$-complex plane. Moreover, the set $\mathbb{C}_J$ is defined $\mathbb{C}_J = \{x + Jy : x, y \in \mathbb{R}, J^2 = p, p \in [-1,0,1]\}$ such that $\mathbb{C}_J \subseteq \mathbb{C}_p$. For $p < 0$, $\mathbb{C}_p$ is called elliptical complex, for $p = 0$, $\mathbb{C}_p$ is called parabolic complex, and for $p > 0$, $\mathbb{C}_p$ is called hyperbolic complex number systems.

In this study, we firstly give the basic notations of the $p$-complex plane $\mathbb{C}_p$. Then, we introduce the one-parameter planar motions in $p$-complex plane $\mathbb{C}_J$, such that $\mathbb{C}_J \subseteq \mathbb{C}_p$. These motions correspond the one-parameter motions in affine Cayley-Klein planes. We examine this motion theory with aspects of complex motions. Besides, we discuss the relations between absolute, relative, sliding velocities (accelerations) and pole curves under the motions $\mathbb{C}_J/\mathbb{C}_J'$.

**Keywords:** Generalized complex number plane, complex-type numbers, one-parameter planar motion, kinematics.

**2010 AMS Classification:** 53A17, 53A35

**References:**


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A New Approach to Motions and Surfaces with Zero Curvatures in Lorentz 3-Space

Esma DEMİR ÇETİN\(^1\) and Yusuf YAYLI\(^2\)

In this work we search for the surfaces with zero curvatures in Lorentz 3-space, whose generating curve is a graph of a polynomial under homothetic motion groups. We study with the generating curves\(\alpha(s) = (f(s), 0, g(s)),\)\(\alpha(s) = (f(s), g(s), 0),\)\(\alpha(s) = (f(s), g(s), f(s))\) depending on the casual character of the axis. (Timelike axis, spacelike axis, lightlike axis respectively.) First of all we see that the degree of the polynomials must be equal for zero curvatures. We show that, distinct from the helicoidal motion groups, these surfaces generated by graph of polynomials don’t have to be ruled surfaces for zero curvatures.

**Keywords:** Gauss curvature, mean curvature, umbilic points, Lorentz space, Homothetic motion

**2010 AMS Classification:** 53B30, 53C50.

**References:**
Chen-Ricci and Wintgen Inequalities for Statistical Submanifolds of Quasi-Constant Curvature

Hülya Aytimur¹, Cihan Özgür²

We define statistical manifold of quasi-constant curvature and give an example. We find Chen-Ricci inequalities, generalized Wintgen inequality for submanifolds in a statistical manifold of quasi-constant curvature.

Keywords: Statistical submanifold \ Chen-Ricci Inequalities \ Wintgen Inequality \ quasi-constant curvature

2010 AMS Classification: 53C40, 53B05, 53B15, 53C05, 53A40

References:

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One-Parameter Homothetic Motion on the Galilean Plane

Mücahit Akbıyık¹, Salim Yüce²

In this paper, we will define one-parameter homothetic motion on the Galilean Plane. The velocities, accelerations and pole points of the motion will be analysed.

Keywords: Galilean(Isotropic) Plane, Kinematics

2010 AMS Classification: 53A17

References:

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Comparison of encryption and decryption algorithms through various approaches

Murat Sari\(^1\), Meliha İpek Bulut\(^2\), İltür Beren Kanpak\(^2\)

The aim of this paper is to produce computer codes of fundamental encryption and decryption algorithms in a comparison way through various approaches such as Substitution, Affine, Vigenere, Caesar and RSA. Performances of the corresponding approaches have been deeply investigated by comparing the usage RAM and CPU times. The codes for the encryption and decryption algorithms are written in C#. General summary of cryptography has also been presented. Effects and performances of computer codes of encryption and decryption algorithms for each one of the methods Affine, Vigenere, Caesar, RSA, Substitution have been compared. For the encryption algorithms, the RSA has been seen to be the best for performance time. For the encryption algorithms, the Affine has been seen to be the best for CPU. For the decryption algorithms, the Substitution has been seen to be the best for performance time. Note also that, for the decryption algorithms, the RSA has been seen to be the best for CPU. Increasing the encryption speed by changing the encryption algorithms is an open problem. Future work can focus on this problem.

**Keywords:** Cryptology, Encryption algorithms, Decryption algorithms, Substitution, Affine, Vigenere, Caesar, RSA.

**2010 AMS Classification:** 14G50, 11T71

**References:**


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Surfaces with Constant Slope and Tubular Surfaces

Çağla RAMİS ¹ and Yusuf YAYLI ²

Tubular surfaces can be characterized as a subfamily of canal surfaces with the constant radius. In this study, we develop the endowed reduced definition of tubular surface and give the new general parameterization by non perpendicular circle along the base curve. Moreover, the advantage of new characterization is to yield a tubular surface without singular points. In accordance with this purpose, we also focus to eliminate singular points by the location of circle which moves along the base curve of surface. Mathematical description of these surfaces enables the relation with constant slope surfaces and the creation of their modeling on computer.

Keywords: Canal surface, tubular surface, surface with constant slope, singularity.

2010 AMS Classification: 53A05

References:

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New Contributions to Fixed-Circle Results on S-Metric Spaces

Ufuk ÇELİK¹, Nihal YILMAZ ÖZGÜR², Nihal TAŞ³

Recently, it has been given some fixed-circle theorems on metric and S-metric spaces. In this talk, we present some new fixed-circle theorems on S-metric spaces. We give new examples of S-metrics and investigate some relationships between circles on metric and S-metric spaces. Then we investigate some existence and uniqueness conditions for fixed circles of self-mappings.

Keyword(s): Fixed circle / fixed-circle theorem / existence theorem / uniqueness theorem / S-metric


Reference(s):
6. Özgür, N.Y. and Taş, N. Some fixed circle theorems on S-metric spaces with a geometric viewpoint, submitted for publication.
On The Parallel Ruled Surfaces With B-Darboux Frame

Mustafa Dede¹, Hatice TOZAK², Cumali Ekici³

In this paper, the parallel ruled surfaces with B-Darboux frame are introduced in Euclidean 3-space. Then some characteristic properties of the parallel ruled surfaces with B-Darboux frame such as developability, striction point and distribution parameter are given in $E^3$.

**Keyword(s):** Parallel ruled surface/ Darboux frame/ Surfaces.

**2010 AMS Classification:** 53A05, 53A15, 53R25

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An ANFIS Perspective for the diagnosis of type II diabetes

Murat Kirişci¹, M. Ubeydullah Saka²

An adaptive network is a multilayer feed forward network in which each node performs a particular function (node function) on incoming signals as well as asset of parameters pertaining to this node. Fuzzy inference systems are the fuzzy rule based systems which consists of a rule base, database, decision making unit, fuzzification interface and a defuzzification interface. By embedding the fuzzy inference system into the framework of adaptive networks, a new architecture namely Adaptive neuro fuzzy inference system (ANFIS) is formed which combines the advantages of neural networks and fuzzy theoretic approaches.

In this study ANFIS is presented for the diagnosis of diabetes diseases. The ANFIS classifier is used to diagnose diabetes disease when six features defining diabetes indications are used as inputs. The proposed ANFIS model is then evaluated and its performance is reported. We are able to achieve significant improvement in accuracy by applying the ANFIS model. Finally, some conclusions are drawn concerning the impacts of features on the diagnosis of diabetes disease.

Keyword(s): diabetes, fuzzy logic, adaptive neuro-fuzzy inference system (ANFIS)

2010 AMS Classification: 68T05, 92C50, 03E72

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Transitive Operator Algebras and Hyperinvariant Subspaces

Elif Demir

In this work, I deal with the transitive and localizing operator algebras. Also I investigate hyperinvariant subspaces.

Keyword(s): Transitive algebra, localizing algebra, hyperinvariant subspace.

2010 AMS Classification: 47L10, 47L45, 47A15

References:
Reformulation of Compressible Fluid Equations in Terms of Biquaternions

Süleyman Demir¹, Murat Tanıştı¹, Mustafa Emre Kansu²

In relevant literature, although Maxwell’s equations of electromagnetism have been expressed in many mathematical forms, the same is not true for their analogous equations in fluid mechanics. In this work, a reformulation is proposed based on biquaternions for the Maxwell type equations of compressible fluids stimulating the biquaternionic generalization of electric and magnetic fields in electromagnetism. After reviewing the analogy between the structure of electrodynamics and fluid dynamics, the biquaternionic expressions of the fluid Maxwell equations have been derived. Furthermore, the field and wave equations for fluids have been presented in a compact and simple way.

Keywords: Biquaternion, fluid equations, Maxwell equations, field equations

2010 AMS Classification: 76A02, 76W05, 11R52

References:

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On The Trace Formula for a Differential Operator of Second Order with Unbounded Operator Coefficients

Erdal GÜL\(^1\) and Duygu ÜÇÜNCÜ\(^2\)

We investigate the spectrum of a differential operator of second order with unbounded operator coefficients and two terms and we calculate the trace of this operator.

**Keywords:** Hilbert Space, Self-adjoint operator, Kernel operator, Spectrum, Essential spectrum, Resolvent.

**2010 AMS Classification:** 47A10, 34L20, 34L05

**References:**

Practical Stability Analyses of Nonlinear Fuzzy Dynamic Systems of Unperturbed Systems with Initial Time Difference

Mustafa Bayram Gücen¹, Coşkun Yakar²

In this work, we have investigated the practical stability of fuzzy differential systems of unperturbed systems and we have established a comparison result. Some practical stability theorems is presented; in the last section, we have a comparison result in practical stability of fuzzy differential systems of unperturbed systems via a scalar differential equation.

**Keyword(s):** Initial time difference, practical stability, fuzzy differential equations

**2010 AMS Classification:** 34D10, 34D99

**Reference(s):**
A Numerical Scheme for Solving Nonlinear Fractional Differential Equations in The Conformable - Derivative

Sebahat Ebru DAŞ¹, Sertan ALKAN²

Fractional Calculus is a field that involves noninteger order differential and integral operators. The history of fractional calculus dates back to the end of the 17th century. In 1695, half-order derivative was mentioned in a letter from L'Hopital to Leibniz [1]. Since then, many mathematicians have contributed to the development of fractional calculus. Therefore, many definitions for the fractional derivative are available. The most popular definitions are Riemann-Liouville and Caputo. Additionally, recently Khalil et at. [2] introduced a new definition of fractional derivative called the Conformable Fractional Derivative.

In our work, Sinc-Collocation Method is presented to obtain the approximate solution of the fractional order boundary value problem with variable coefficients in the following form

\[ \mu_2(x)y''(x) + \mu_\alpha(x)y^{(\alpha)}(x) + \mu_1(x)y'(x) + \mu_\beta(x)y^{(\beta)}(x) + \mu_0(x)y(x) + n(x)y^n(x) = f(x) \]

with boundary conditions

\[ y(a) = 0 \quad , \quad y(b) = 0 \]

where \( y^{(\alpha)} \) and \( y^{(\beta)} \) are the conformable fractional derivative for \( 1 < \alpha \leq 2 \) and \( 0 < \beta \leq 1 \).

Keywords: Nonlinear differential equations, conformable –derivative

2010 AMS Classification: 34A08

Reference(s):
1. Samko S.G., Kilbas A.A., Marichev O.I., Fractional Integrals and Derivatives, Gordon and Beach, Yverdon, 1993
On Locally Convex Solid Riesz Spaces

Fatma ÖZTÜRK ÇELİKER and Pınar ALBAYRAK

An ordered vector space $E$ is called a Riesz space if every pair of vectors has a supremum and an infimum. A locally convex (solid) topology on a vector space is a linear topology that has a base at zero consisting of convex (solid) sets. A subset $S$ of Riesz space $E$ is said to be solid if $|u| \leq |v|$ and $v \in S$ imply $u \in S$. A linear topology $\tau$ on a Riesz space $E$ that is at the same time locally solid and locally convex will be called a locally convex-solid topology. A locally convex-solid Riesz space $(E, \tau)$ is a Riesz space $E$ equipped with a locally convex-solid topology $\tau$.

On this study we consider invariant ideals on locally convex solid Riesz spaces for positive operators.

Keyword(s): Invariant ideals, locally convex solid Riesz Spaces

2010 AMS Classification: 47A15

References:


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On Weakly Compact-Friendly Operators

Pınar Albayrak¹ and Fatma Öztürk Çeliker¹

A positive operator $B : E \rightarrow E$ is said to be weakly compact-friendly if there exists a positive operator in the commutant of $B$ dominates a non-zero operator which in turn is dominated by positive weakly compact operator. That is, $B$ is weakly compact-friendly if and only if there exists three non-zero operators $R, C, K : E \rightarrow E$ with $R, K$ positive and $K$ weakly compact such that

$$RB = BR, \quad |Cx| \leq R(|x|), \quad \text{and} \quad |Cx| \leq K(|x|)$$

for each $x \in E$.

On this talk we generalized some well-known results of weakly compact-friendly operators on Banach lattices.

Keyword(s): Invariant ideals, invariant subspaces, weakly compact-friendly operators

2010 AMS Classification: 47B65, 47A15

References:


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Hirota Type Discretization Of Clebsch Equations

Murat Turhan

The equations of motion of a rigid body in an ideal fluid is given by the following system:

\[
\begin{align*}
\dot{x} &= x \times \frac{\partial H}{\partial p} \\
\dot{p} &= x \times \frac{\partial H}{\partial x} + p \times \frac{\partial H}{\partial p}
\end{align*}
\]

where \( H \in C^\infty(\mathbb{R}^6, \mathbb{R}) \) is a quadratic polynomial in \( x \) and \( p \).

Applying bilinear method and using the gauge invariance and the time reversibility of the equations, we get gauge-invariant bilinear difference equations. Finally, we derive the explicit discrete system by considering Hirota bilinear transformation method and present sufficient number of the discrete conserved quantities for integrability.

**Keywords:** Clebsch system, discretization, bilinear form, Gröbner basis

**2010 AMS Classification:** 70H99

**References:**


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An h-deformation of the Superspace R(1|2) via a contraction

Salih Çelik¹, Sultan A. Çelik¹ and Fatma Bulut²

A deformation of classical matrix (super) groups can be made using some facts known from the classical. One way to obtain a deformation of classical (super) groups is to make a deformation of (super) spaces first [1].

The one-parametric $h$-deformation of the algebra of coordinate functions on the superspace $R(1|2)$ via a contraction of the quantum superspace $R_q(1|2)$ is presented [2]. An interesting case is that the deformation parameter $h$ is Grassmann number.

It is well known that a matrix $T$ in the supergroup $GL(1|2)$ defines the linear transformation $T : R_h(1|2) \rightarrow R_h(1|2)$. As a result of this we have $TX = X' \in R_h(1|2)$. So, the elements of the matrix $T$ fulfill some relations. The bi-algebra structure of $GL_h(1|2)$ is discussed.

**Keywords:** Quantum superspace, q-deformation, h-deformation, quantum supergroup, Hopf superalgebra.

**2010 AMS Classification:** 17B37, 81R60

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7- Celik, S., Bicovariant differential calculus on the superspace $\mathbb{R}q(1j2)$, J. Alg. and Its Appl. 15, No.9 (2016), 1650172 (17 pages)

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Euler-Savary’s Formula on Dual Plane

Mücahit Akbıyık¹, Salim Yüce²

In this work, one three Dual planes, of which two of them are moving and the other one is fixed, are considered during the one parameter motion. In each motion; the velocities, the relation between the velocities and the rotation poles were calculated. In addition, Euler-Savary formula, which gives the relationship between the curvature of pole curves and trajectory curve, were given by two different methods.

**Keywords:** Dual Plane, Euler Savary’s Formula, Kinematics.

**2010 AMS Classification:** 53A17,53A35, 53A40.

**References:**


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21. Masal, M., Tosun, M., Pirdal, A. Z., Euler Savary formula for the one parameter motions in the complex plane

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Mistakes and Misconceptions Regarding to Natural Numbers on Secondary Mathematics Education

Ayten Özkan¹ and Erdoğan Mehmet Özkan¹

Mistakes and misconceptions regarding to natural numbers of 12th class students and whether these mistakes and misconceptions demonstrated any significant difference depending on the gender has been investigated in this study. This study was carried out with 60 students at 12th class who have being educated in Anatolian High Schools located in Istanbul City Fatih Province in the education training year of 2017-2018 after completing of the natural number subject. Cronbach Alpha coefficient reliability of the Diagnosis Test was found as 0,90. An expert opinion was obtained for the validity. The SPSS 15 pack program was used in order to solve the data obtained by Diagnosis Test which composed by open-ended questions. Qualitative and quantitative researching methods were utilized in this study. The answers given by the students were examined individually and the answers of the students were evaluated in categories such as “correct”, “mistake”, “empty” and “misconception”, then distribution of these students’ answers into percentage and frequency categories were determined. Also samples within all determined mistakes and misconceptions transferred into the computer via scanner and were submitted in findings. At the end of the investigation, it has been determined that students had a lot of mistake and misconceptions regarding to natural number, features belong to exponentiation, base arithmetic, prime numbers, relative prime numbers, prime factorization of a natural number, positive dividends of a natural number and factorial. Also these mistakes and misconceptions were determined as not demonstrating a significant difference depending on the genders.

Keywords: Mathematics teaching, misconception, common mistakes, natural numbers

2010 AMS Classification: 97C10, 97C70

References:


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On a Fuzzy Application of the Particulate Matter Estimation

Filiz Kanbay\textsuperscript{1}, Nurten Vardar\textsuperscript{2}

In this study, Particulate matter PM from transit vessels passing through the Bosphorus which connects Black sea and sea of Marmara with the length 12 sea knot are calculated by using fuzzy inference system in MATLAB. Total particulate matters from ships are expressed surfaces and these results allow the analysis of the data gross tone and the type of ships.

\textbf{Keywords:} Particulate matter, fuzzy, surface, ship

\textbf{2010 AMS Classification:} 65D18, 68T27, 93B99

\textbf{References:}


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